

Exam 1 Review

(1.) Find the indicated limit. JUSTIFY EACH STEP.

(a.) $\lim_{x \rightarrow 2} \frac{2x + 1}{5 - 3x}$

(b.) $\lim_{w \rightarrow -2} \sqrt{-3w^3 + 7w^2}$

(2.) Find the indicated limits:

(a.) $\lim_{h \rightarrow 0} \frac{\cos(2 + h) - \cos(2)}{h}$

(b.) $\lim_{\theta \rightarrow 0} \frac{\sin 3\theta}{2\theta}$

(c.) $\lim_{t \rightarrow 3^-} \frac{t^2}{9 - t^2}$

(d.) $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$

(e.) $\lim_{x \rightarrow \infty} \sqrt{3x^2 + 5} - \sqrt{3x^2 - 2}$

(f.) $\lim_{x \rightarrow 4} \frac{x - 4}{\sqrt{x} - 2}$

(3.) Show that the given equation has at least one real solution.

(a.) $t^4 + t^2 - 7t = -4$

(b.) $\sin \theta + \cos \theta = \theta^2$

(4.) Let $f(x) = \frac{x+1}{x^2-1}$. Find the indicated limits:

(a.) $\lim_{x \rightarrow 1^-} f(x)$

(b.) $\lim_{x \rightarrow 1^+} f(x)$

(c.) $\lim_{x \rightarrow \infty} f(x)$

(d.) $\lim_{x \rightarrow -1} f(x)$

(5.) For the given $f(x)$, find the values of a and b which make $f(x)$ continuous everywhere.

(a.) $f(x) = \begin{cases} 2x - 1 & \text{if } x < 1 \\ ax + b & \text{if } 1 \leq x < 2 \\ -x & \text{if } x \geq 2 \end{cases}$

(b.) $f(x) = \begin{cases} 2x^2 + 1 & \text{if } x < -1 \\ ax + b & \text{if } -1 \leq x < 1 \\ x + 1 & \text{if } x \geq 1 \end{cases}$

(6.) Prove the given limit using an $\epsilon - \delta$ proof.

(a.) $\lim_{x \rightarrow 2} \sqrt{3x + 1} = \sqrt{7}$

(b.) $\lim_{x \rightarrow 2} x^2 - 3x = -2$