

Quiz 2

Instructions: Each question is worth 5 points. You may use any notes or books but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. Make sure to write clearly and justify your answers.

(1.) Let $A = \begin{pmatrix} 2 - \lambda & -3 & 1 \\ 1 & -2 - \lambda & 1 \\ 1 & -3 & 2 - \lambda \end{pmatrix}$. Find all values of λ such that $\det(A) = 0$. DO NOT USE MATLAB

(2.) Suppose that A is non-singular matrix. Show that $\det(A^{-1}) = \frac{1}{\det(A)}$.

(3.) Suppose that A is an $n \times n$ matrix and α is a scalar. Show that $\det(\alpha A) = \alpha^n \det(A)$. (Hint: $\alpha A = \alpha I_n A$)

(4.) Let $V = \begin{pmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{pmatrix}$. V is known as the 3×3 Vandermonde matrix.

(a.) Show that $\det(V) = (x_2 - x_1)(x_3 - x_1)(x_3 - x_2)$. (Hint: use row reduction)

(b.) Under what conditions on x_1, x_2 and x_3 is V non-singular?

(5.) Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 9 & \alpha \\ 1 & \alpha & 3 \end{pmatrix}$. Find all the values for α which make A singular.

(6.) Let A be an $n \times n$ matrix. Show that $\det(P_{12}A) = -\det(A)$. (Hint: expand A along row 1 and $P_{12}A$ along row 2 and compare)