

**Quiz 4**

**Instructions:** This quiz is worth a total of 20 points, and the value of each question is listed with each question. You may use any notes or books but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. Make sure to write clearly and justify your answers.

(1.)(4 pts.) Determine if the given vectors are linearly independent in  $V$  where:

- (a.)  $\{(3, -1, -4)^T, (3, -3, -1)^T, (12, -2, -19)^T\}$  in  $V = \mathbb{R}^3$ .
- (b.)  $\{x^2 - 1, x + 1, x - 3\}$  in  $V = \mathbb{P}_3$ .
- (c.)  $\{e^{2x}, e^{2(x-1)}\}$  in  $V = C[-1, 1]$ .
- (d.)  $\{\sin mx, \cos nx\}$  in  $V = C[-\pi, \pi]$ .

(2.)(4 pts.) Find a basis for the subspace  $S$  of  $V$  where:

- (a.)  $S = \text{Span}((-3, -2, 0, 2)^T, (-5, 2, -4, 8)^T, (-2, -4, 2, -1)^T, (1, -2, 2, -3)^T)$  and  $V = \mathbb{R}^4$ .
- (b.)  $S = \{p(x) \in \mathbb{P}_4 \mid p(2) = 0 \text{ and } p(-1) = 0\}$  and  $V = \mathbb{P}_4$ .
- (c.)  $S = \{A \in \mathbb{R}^{3 \times 3} \mid A \text{ is lower triangular}\}$  and  $V = \mathbb{R}^{3 \times 3}$ .
- (d.)  $S = \text{Span}(1, \cos^2 x, \cos 2x)$  and  $V = C[-\pi, \pi]$ .

(3.)(12 pts.) For the given vector space  $V$  and the subset  $S$  do the following:

- (i.) Show that  $S$  is a subspace of  $V$ .
  - (ii.) Find a basis for  $S$ .
  - (iii.) Determine  $\dim(S)$ .
- (a.)  $V = \mathbb{R}^4$  and  $S$  is the set of vectors  $(x_1, x_2, x_3, x_4)^T$  with  $x_3 = 3x_1 + x_2$  and  $x_4 = x_1 - 2x_2$ .
  - (b.)  $V = \mathbb{R}^{2 \times 2}$  and  $S$  is the set of  $2 \times 2$  matrices  $A = (a_{ij})$  with  $a_{11} - a_{22} = 0$ .
  - (c.)  $V = \mathbb{P}_4$  and  $S$  is the set of polynomials  $p(x)$  in  $V$  such that  $p'(2) = 0$ .
  - (d.)  $V = \mathbb{R}^{3 \times 3}$  and  $S$  is the set of  $3 \times 3$  upper triangular matrices  $A$  with  $A^3 = \mathbf{0}$ . (hint: Find a typical vector first.)