Math 571
Spring 2008

Quiz 3

Instructions: This quiz is worth a total of 10 points, and the value of each question is listed with each question. You may use any notes or books but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. Make sure to write clearly and justify your answers.

(1.)(4.5 pts.) Determine if S is a subspace of the vector space V where:

(a.) $V = \mathbb{R}^3$ and $S = \{(x_1, x_2, x_3)^T \in \mathbb{R}^3 \mid x_2 = x_1 + 2x_3\}$

(b.) $V = \mathbb{P}_4$ and $S = \{p(x) \in \mathbb{P}_4 \mid p(x)$ has al least one real root$\}$

(c.) $V = C[-1, 1]$ and $S = \{f(x) \in C[-1, 1] \mid f(-1) = 2f(1)\}$

(d.) $V = \mathbb{R}^3$ and $S = \{(x_1, x_2, x_3)^T \in \mathbb{R}^3 \mid x_1 = x_2$ or $x_1 = -x_2\}$

(e.) $V = \mathbb{R}^{3\times 3}$ and $S = \{A \in \mathbb{R}^{3\times 3} \mid A^T = A\}$

(f.) $V = C^2[-1, 2]$ and S is the set of functions $f(x)$ such that $f''(0) \leq 0$.

(g.) $V = C^2[-1, 2]$ and S is the set of functions $f(x)$ such that $f''(x) + 2f'(x) + 2f(x) = 0$.

(h.) $V = \mathbb{R}^3$ and $S = \{(x, y, z)^T \mid x + y = z^2\}$

(i.) $V = \mathbb{R}^{4\times 4}$ and S is the set of 4×4 singular matrices.

(2.)(3.5 pts.) Determine if S is a spanning set for V where:

(a.) $V = \mathbb{R}^3$ and $S = \{(1, 1, 0)^T, (3, 2, 3)^T, (-1, 0, -3)^T\}$

(b.) $V = \mathbb{P}_3$ and $S = \{1 - x^2, 1 + x, x^2 - x + 1\}$

(c.) $V = \mathbb{R}^{2\times 2}$ and $S = \left\{ \begin{pmatrix} 1 & 0 \\ -1 & 7 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ 3 & 5 \end{pmatrix}, \begin{pmatrix} -2 & 4 \\ 6 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} \right\}$

(3.)(2 pts.) Suppose that U and V are subspaces of the vector space W. Prove that the following are also subspaces of W:

(a.) $U + V = \{\mathbf{w} = \mathbf{u} + \mathbf{v} \mid \mathbf{u} \in U$ and $\mathbf{v} \in V\}$

(b.) $U \cap V = \{\mathbf{w} \in W \mid \mathbf{w} \in U$ and $\mathbf{w} \in V\}$