

Review 1

(1.) Consider the following system of linear equations:

$$\begin{aligned} 3x_1 - 2x_2 + 4x_3 + 3x_4 - x_5 &= 17 \\ 2x_1 + 4x_3 + x_4 + 2x_5 &= 8 \\ -2x_1 + 2x_2 - 2x_3 - x_4 + 2x_5 &= -10 \\ -2x_1 - 2x_2 - 6x_3 + 2x_4 - 6x_5 &= 0 \end{aligned}$$

- (a.) Find the augmented matrix of this system.
- (b.) Find the reduced row echelon form of the matrix in (a.).
- (c.) Is this system consistent? If so, find all solutions.

(2.) Given the augmented matrix  $A = \left( \begin{array}{ccc|c} 1 & -1 & 3 & 1 \\ 3 & 2 & -2 & -2 \\ -2 & 7 & \alpha & \beta \end{array} \right)$ , find all values of  $\alpha$  and  $\beta$  such that the corresponding linear system has:

- (a.) No solutions.
- (b.) A unique solution.
- (c.) Infinitely many solutions.

(3.) Determine if  $S$  is a subspace of  $V$  where:

- (a.)  $V = \mathbb{R}^{2 \times 2}$  and  $S$  is the set of  $2 \times 2$  matrices  $A$  with  $\det(A) = 0$ .
- (b.)  $V = \mathbb{R}^{2 \times 2}$  and  $S$  is the set of  $2 \times 2$  upper triangular matrices.
- (c.)  $V = \mathbb{R}^2$  and  $S = \{(x_1, x_2)^T \mid |x_1| = |x_2|\}$ .
- (d.)  $V = \mathbb{P}_5$  and  $S$  is the set of all polynomials  $p(x)$  in  $V$  such that  $p(1) = 0$ .
- (e.)  $V = C[-1, 1]$  and  $S$  is the set of odd functions in  $V$ .

(4.) Suppose  $A = \begin{pmatrix} 2 & 1 & 3 \\ 0 & 0 & 7 \\ 4 & 0 & 11 \end{pmatrix}$  and  $B = \begin{pmatrix} 2 & 1 & 3 \\ 0 & -2 & 5 \\ 0 & 0 & 1 \end{pmatrix}$

- (a.) Find three elementary matrices  $\mathcal{E}_1$ ,  $\mathcal{E}_2$ , and  $\mathcal{E}_3$  such that  $\mathcal{E}_3\mathcal{E}_2\mathcal{E}_1A = B$ .
- (b.) Compute  $\det(A)$  using the results of part (a.). DO NOT USE MATLAB.

(5.) Find an  $LU$ -factorization for  $A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 7 \\ -2 & 4 & 1 \end{pmatrix}$ . DO NOT USE MATLAB

(6.) True or False. You do not need to explain your answer.

- (a.) If  $A$  and  $B$  are  $n \times n$  matrices, then  $\det(A + B) = \det(A) + \det(B)$ .
- (b.) If  $A$  and  $B$  are  $n \times n$  matrices, then  $AB = BA$ .
- (c.) Suppose  $A\mathbf{x} = \mathbf{b}_1$  has a unique solution, then it is possible for  $A\mathbf{x} = \mathbf{b}_2$  to have more than one solution.
- (d.) An underdetermined system of linear equations is always consistent.
- (e.) Every  $n \times n$  elementary matrix is invertible.
- (f.) If  $A$  is an  $n \times n$  invertible matrix, then  $A$  can be row reduced to  $I_n$ .
- (g.) If  $A$  is an  $n \times n$  matrix and  $\alpha$  is a scalar, then  $\det(\alpha A) = \alpha \det(A)$ .

(7.) Prove the following:

- (a.) If  $A$  is an invertible, then  $\det(A^{-1}) = \det(A)^{-1}$ .
- (b.) If  $A$  is an invertible, then  $A\mathbf{x} = \mathbf{b}$  has a unique solution for each  $\mathbf{b}$ .
- (c.) If  $A$  is an invertible, then  $A = \mathcal{E}_1\mathcal{E}_2 \cdots \mathcal{E}_k$  where  $\mathcal{E}_1, \mathcal{E}_2, \dots, \mathcal{E}_k$  are some elementary matrices.
- (d.) If  $A\mathbf{x}_1 = \mathbf{b}$  and  $A\mathbf{x}_2 = \mathbf{b}$ , then  $(\mathbf{x}_1 - \mathbf{x}_2)$  is a solution to  $A\mathbf{x} = \mathbf{0}$ .