

(1.) Let $A = \begin{pmatrix} 3 & 6 & 3 & 3 & 6 & 1 \\ 5 & 10 & 4 & 2 & -3 & 6 \\ 7 & 14 & 2 & 0 & -11 & 7 \\ 1 & 2 & 0 & 0 & -1 & 2 \end{pmatrix}$.

- (a.) Find a basis for the row space of A .
- (b.) Find a basis for the column space of A .
- (c.) Find a basis for the nullspace of A . DO NOT USE MATLAB.
- (d.) The rank of A .

(2.) Suppose that $\mathbf{x}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\mathbf{x}_2 = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}$, $\mathbf{x}_3 = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ and $\mathbf{x}_4 = \begin{pmatrix} 2 \\ 2 \\ 8 \end{pmatrix}$. Find a basis for $\text{Span}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4)$.

Note: there is more than one answer.

(3.) Determine if $1, e^x$ and $\cos x$ are linearly independent in $C[0, 1]$.

(4.) Find a basis for the subspace S of V where:

- (a.) $V = \mathbb{R}^4$ and $S = \{(a - b + c, a + c, a + 2b - c, b - 3c)^T \mid a, b, c \text{ are real numbers}\}$.
- (b.) $V = C[0, 1]$ and $S = \text{Span}(1, \sin 2x, \sin x \cos x)$.
- (c.) $V = \mathbb{P}_4$ and S is the set of all polynomials $p(x)$ in V with $p(0) = 0$ and $p(1) = 0$.

(5.) Let $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and let $S = \{B \in \mathbb{R}^{2 \times 2} \mid AB = BA\}$.

- (a.) Show that S is a subspace of $\mathbb{R}^{2 \times 2}$.
- (b.) Find a basis for S .

(6.) Let $x_1 = 1, x_2 = 2, x_3 = -1$ and $x_4 = -2$. Then

$$\langle p(x), q(x) \rangle = p(x_1)q(x_1) + p(x_2)q(x_2) + p(x_3)q(x_3) + p(x_4)q(x_4)$$

defines an inner product on \mathbb{P}_4 . If $p(x) = x^3 - 6x^2 + 2$ and $q(x) = x^3 + x - 1$, find the following with respect to THIS inner product:

- (a.) $\langle p(x), q(x) \rangle$, $\|p(x)\|$, $\|q(x)\|$,
- (b.) the orthogonal projection of $p(x)$ onto the line spanned by $q(x)$ and
- (c.) the angle between $p(x)$ and $q(x)$.

(7.) True or False?

- (a.) A linearly independent set can not contain $\mathbf{0}$.
- (b.) If \mathbf{x} and \mathbf{y} are vectors in the inner product space V , then $\|\mathbf{x} + \mathbf{y}\|^2 = \|\mathbf{x}\|^2 + \|\mathbf{y}\|^2$.
- (c.) If A is an $m \times n$ matrix, then $\dim(\text{Col}(A)) + \dim(\text{N}(A)) = m$.
- (d.) If A is an $m \times n$ matrix, then $A\mathbf{x} = \mathbf{b}$ is consistent if and only if \mathbf{b} is in the column space of A .
- (e.) If A is a singular $n \times n$ matrix, then the columns of A form a basis for \mathbb{R}^n .
- (f.) A spanning set can never be linearly independent.

(8.) Prove the following:

(a.) Let S be the subset of $\mathbb{R}^{n \times n}$ consisting of matrices A such that $A^T = A$. Show that S is a subspace of $\mathbb{R}^{n \times n}$. (The matrices in S are called symmetric matrices.)

(b.) If $\{\mathbf{x}_1, \dots, \mathbf{x}_n\}$ are linearly independent vectors in a vector space V , then $\{\mathbf{x}_2, \dots, \mathbf{x}_n\}$ do not span V .

(c.) If $\{\mathbf{x}_1, \dots, \mathbf{x}_n\}$ are linearly independent vectors in a vector space V which do not span V , then there is a vector \mathbf{x}_0 in V such that $\{\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_n\}$ is also linearly independent.

(d.) If \mathbf{p} is the orthogonal projection of \mathbf{x} onto the line spanned by \mathbf{y} then \mathbf{p} and $\mathbf{x} - \mathbf{p}$ are orthogonal.