

HW1

March 30, 2007

1. Show the maximum principle for *ENO* scheme or give a counterexample.
ENO scheme for $u_t + u_x = 0$:

$$u_j^{n+1} = u_j^n - \lambda(u_j^n - u_{j-1}^n) + \frac{1}{2}\lambda(\lambda - 1)m \left\{ \begin{array}{l} u_{j+1}^n - 2u_j^n + u_{j-1}^n \\ u_j^n - 2u_{j-1}^n + u_{j-2}^n \end{array} \right\}$$

where $\lambda = \frac{\Delta t}{\Delta x}$ and $m(x, y) =$ smaller magnitude of x and y if $xy > 0$;
otherwise $m = 0$.

maximum principle:

$$\min \{u_{j+1}^n, u_j^n, u_{j-1}^n, u_{j-2}^n\} \leq u_j^{n+1} \leq \max \{u_{j+1}^n, u_j^n, u_{j-1}^n, u_{j-2}^n\}$$

2. Show that the maximum principle is not true if Lax-Wendriff or 2nd upwind method is used.

3. a, b, c, d are real constants. Find condition on a, b, c, d so that the following system is hyperbolic

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_t = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}_x$$