

HW4

April 29, 2009

1. For a system of linear wave equations

$$U_t + AU_x = 0$$

where

$$U = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \quad A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

with initial condition

$$U_0 = \begin{pmatrix} \sin(\pi x) \\ \cos(\frac{1}{2}\pi x) \\ -\sin(2\pi x) \end{pmatrix} \quad \text{if } -1 < x < 1$$

and

$$U_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{otherwise}$$

Write the code to solve this system by the following methods

- (a) 1st Order Upwind
- (b) Lax-Friedrichs
- (c) Lax-Wendroff
- (d) Leap-frog

in the interval $x \in [-2, 6]$ with $N = 800$ up to time $t = 2$. Show movies of U with time interval $\Delta t_m = 0.1$.

Find CFL condition for the problem. Analyze the stability conditions for the above schemes.

2. Ex 4.8-4.10, 6.1-6.3 in the book