MATH 836 WINTER 2010

PARTIAL DIFFERENTIAL EQUATIONS II
HOMEWORK ASSIGNMENTS

Due January 20 1. Evans, p 235 #9.

2. For the first-order equation example, $u_t + u_x = 0$ with data $u(x,0) = f(x) = \sum f_n x^n$, prove directly the convergence of the power series solution. If f has radius of convergence r, what is an estimate for the radius of convergence of the series for the solution?

3. For an equation of any order, in any number of independent variables with Cauchy data:

$$\partial_t^m u(x,t) = G(x,t,D^\alpha u), \quad \partial_t^k u(x,0) = f_k(x), \quad 0 \le k \le m-1,$$

where $D^\alpha u$ represents the collection of partial derivatives of u of total order less than or equal to m, excluding $\partial_t^m u$, derive the reduction to a canonical system.

Due February 19 1. Determine whether the following operators satisfy the Hadamard-Petrowsky condition:

$$\partial_{tt} \pm (\Delta)^2, \qquad \partial_t \pm (\Delta)^2, \qquad \partial_t + \partial_x^m, \qquad \partial_t^m + \epsilon \partial_x.$$

2. Find necessary and sufficient conditions on (real) a and b for the forward H-P condition to hold for $u_{tt} + a u_{tx} + b u_{xx}$.

3. What can you say about the well-posedness of the sideways Cauchy problem for the wave equation $(\partial_{tt} - \Delta)u = 0$:

$$u|_{x_1=0} = f, \qquad u_{x_1}|_{x_1=0} = g, \qquad \text{in } \mathbb{R}^d \times \mathbb{R},$$

for $d = 1$ and $d > 1$?