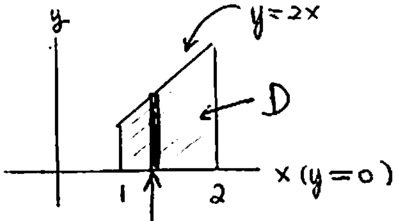
Solutions to Some Problems from Section 15.3

(8)



$$\iint_D \frac{4y}{x^3+2}\,dA = \int_{x=1}^{x=2}\int_{y=0}^{2x} \frac{4y}{x^3+2}\,dy\,dx$$

$$= \int_{x=1}^{x=2} \frac{8x^2}{x^3+2}\,dx$$

$$= \left[\frac{8}{3}\ln(x^3+2)\right]_{x=1}^{2}$$

$$= \frac{8}{3}\ln 10 - \frac{8}{3}\ln 3$$

$$= \frac{8}{3}\ln\frac{10}{3}$$

(use the fact that $\ln a - \ln b = \ln\frac{a}{b}$ provided $a>0$, $b>0$)

$$\int_{y=0}^{y=2x} \frac{4y}{x^3+2}\,dy$$

$$= \left[\frac{2y^2}{x^3+2}\right]_{y=0}^{y=2x}$$

$$= \frac{8x^2}{x^3+2}$$

---

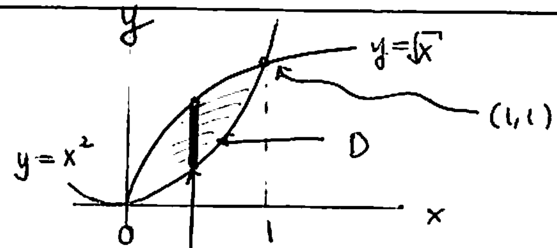
(14)

$$\iint_D (x+y)\,dA$$

$$= \int_{x=0}^{x=1}\int_{y=x^2}^{y=\sqrt{x}} (x+y)\,dy\,dx$$

$$= \int_{x=0}^{x=1}\left(x^{\frac{3}{2}} + \frac{x}{2} - x^3 - \frac{x^4}{2}\right)dx$$

$$= \frac{3}{10}$$

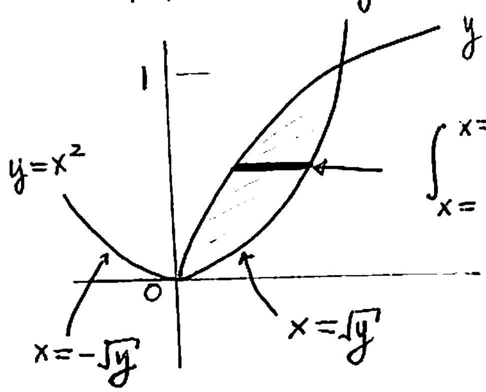


$$\int_{y=x^2}^{y=\sqrt{x}} (x+y)\,dy$$

$$= \left[xy + \frac{y^2}{2}\right]_{y=x^2}^{y=\sqrt{x}}$$

$$= x^{\frac{3}{2}} + \frac{x}{2} - x^3 - \frac{x^4}{2}$$

Alternatively



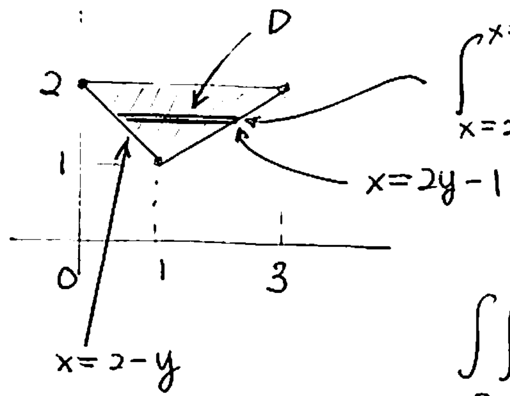
$$\int_{x=y^2}^{x=\sqrt{y}} (x+y)\,dx = y^{\frac{3}{2}} + \frac{y}{2} - y^3 - \frac{y^4}{2}$$

$$\iint_D (x+y)\,dA = \int_{y=0}^{y=1}\int_{x=y^2}^{x=\sqrt{y}} (x+y)\,dx\,dy$$

$$= \int_{y=0}^{y=1}\left(y^{\frac{3}{2}} + \frac{y}{2} - y^3 - \frac{y^4}{2}\right)dy$$

$$= \frac{3}{10}$$

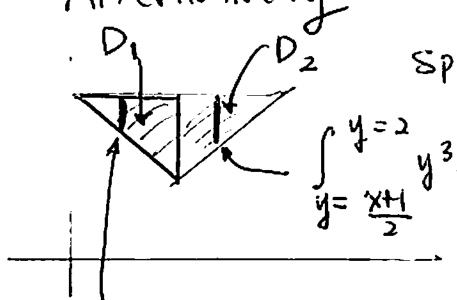
(15)



$$\int_{x=2-y}^{x=2y-1} y^3\,dx = \Big[\, x y^3 \,\Big]_{x=2-y}^{x=2y-1} = 3y^4 - 3y^3$$

$$\iint_D y^3\,dA = \int_{y=1}^{y=2}\int_{x=2-y}^{x=2y-1} y^3\,dx\,dy = \int_{y=1}^{y=2} (3y^4 - 3y^3)\,dy = \frac{147}{20}$$

Alternatively



Split D into D_1 and D_2

$$\int_{y=\frac{x+1}{2}}^{y=2} y^3\,dy = -\frac{x^4}{64} - \frac{x^3}{16} - \frac{3x^2}{32} - \frac{x}{16} + \frac{255}{64}$$

* Note: $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$$\int_{y=2-x}^{y=2} y^3\,dy = \left[\frac{y^4}{4}\right]_{y=2-x}^{y=2} = -\frac{x^4}{4} + 2x^3 - 6x^2 + 8x$$

$$\iint_D y^3\,dA = \iint_{D_1} y^3\,dA + \iint_{D_2} y^3\,dA$$

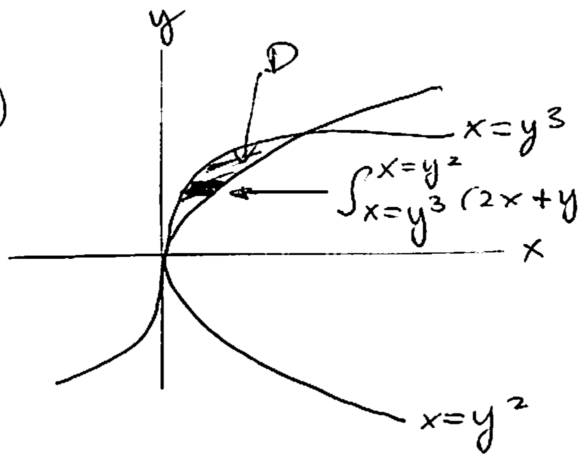
$$= \int_{x=0}^{x=1}\int_{y=2-x}^{y=2} y^3\,dy\,dx + \int_{x=1}^{x=3}\int_{y=\frac{x+1}{2}}^{y=2} y^3\,dy\,dx$$

$$= \int_{x=0}^{1}\left(-\frac{x^4}{4} + 2x^3 - 6x^2 + 8x\right)dx + \int_{x=1}^{3}\left(-\frac{x^4}{64} - \frac{x^3}{16} - \frac{3x^2}{32} - \frac{x}{16} + \frac{255}{64}\right)dx$$

$$= \frac{147}{20}$$

Choosing the right order of integration eases the subsequent computation !!

(20)



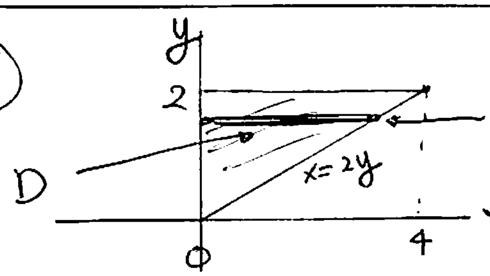
$$\int_{x=y^3}^{x=y^2} (2x+y^2)\,dx = 2y^4 - y^6 - y^5$$

$$\text{Volume} = \iint_D \underbrace{(2x+y^2)}_{\text{"height" of the solid}}\,dA = \int_{y=0}^{y=1}\int_{x=y^3}^{x=y^2} (2x+y^2)\,dx\,dy$$

$$= \frac{19}{210}$$

---

(26)



$$\int_{x=0}^{x=2y} \sqrt{4-y^2}\,dx = 2y\sqrt{4-y^2}$$

$$\text{volume} = \iint_D \underbrace{\sqrt{4-y^2}}_{\text{"height" of the solid}}\,dA = \int_{y=0}^{y=2}\int_{x=0}^{x=2y} \sqrt{4-y^2}\,dx\,dy$$

$$= \int_{y=0}^{y=2} 2y\sqrt{4-y^2}\,dy$$

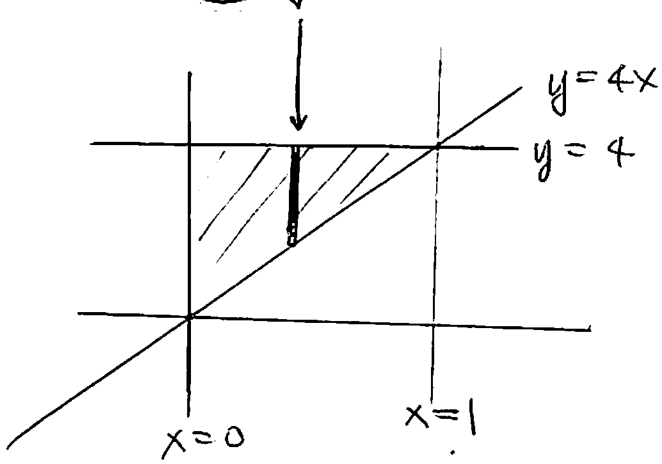
Use substitution $u = 4 - y^2$

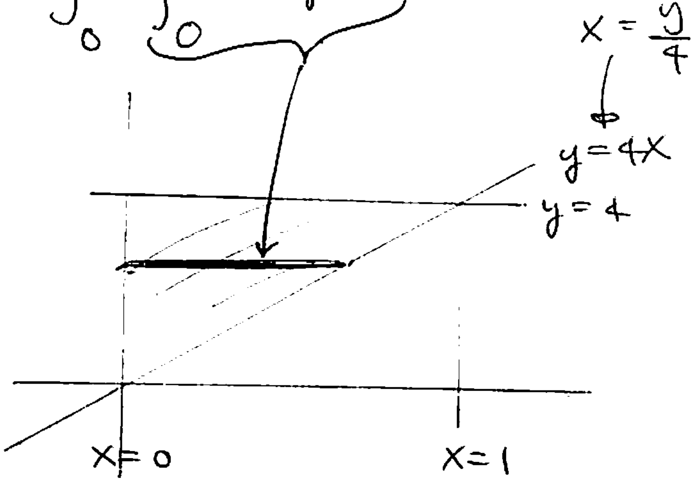
$$= \frac{16}{3}$$

---

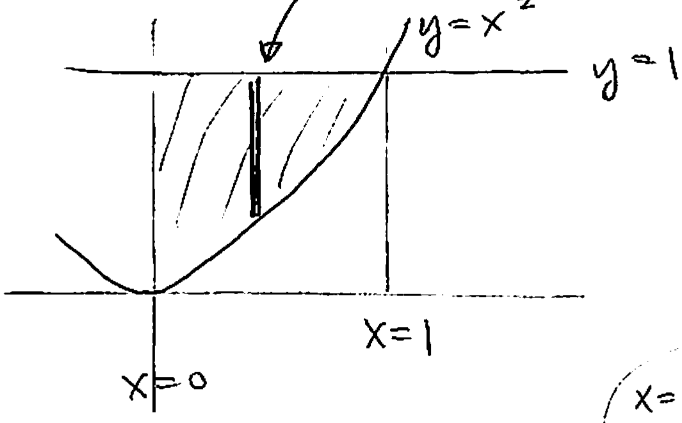
(38)

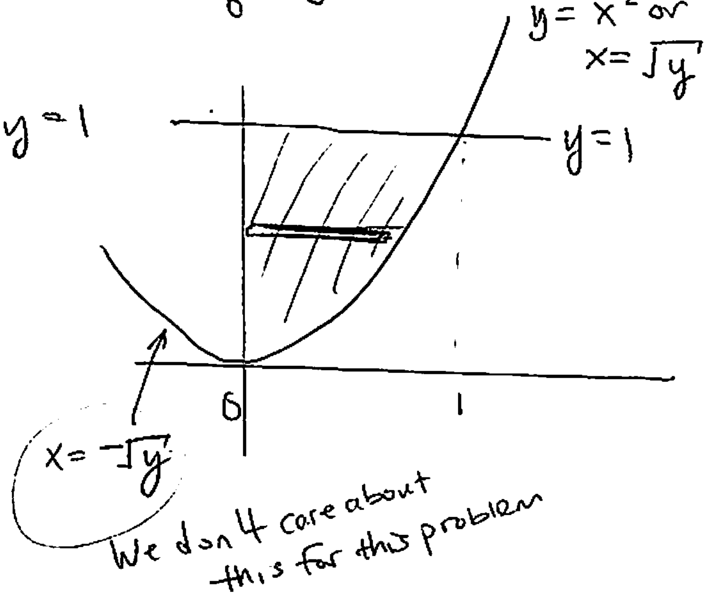
$$\int_0^1 \int_{4x}^{4} f(x,y)\,dy\,dx = \int_0^4 \int_0^{\frac{y}{4}} f(x,y)\,dx\,dy$$





(46) $\int_0^1 \int_{x^2}^1 x^3 \sin(y^3)\,dy\,dx = \int_0^1 \int_0^{\sqrt{y}} x^3 \sin(y^3)\,dx\,dy$





So $\int_0^1 \int_{x^2}^1 x^3 \sin(y^3)\,dy\,dx = \int_0^1 \left[\frac{x^4}{4}\sin(y^3)\right]_{x=0}^{\sqrt{y}} dy$

$= \int_0^1 \frac{y^2}{4}\sin(y^3)\,dy$

$= \left[-\frac{1}{12}\cos(y^3)\right]_0^1 = \frac{1}{12}(1-\cos 1)$