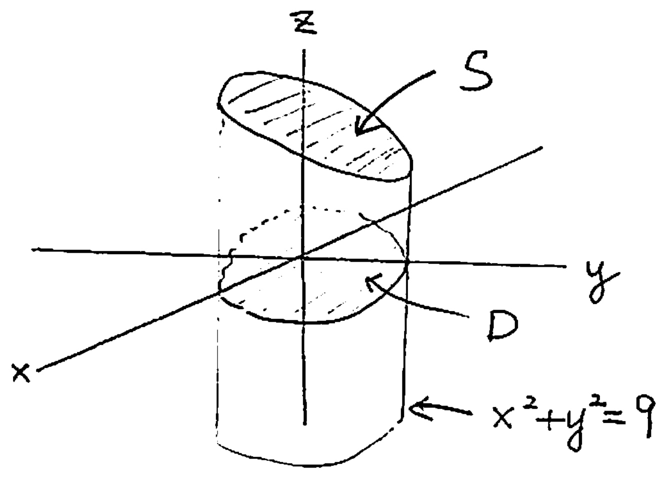
Section 15.6: Surface Area (Problems)

② $z = f(x,y) = 10 - 2x - 5y$

$f_x(x,y) = -2$

$f_y(x,y) = -5$



$$A(S) = \iint_D \sqrt{(f_x)^2 + (f_y)^2 + 1}\, dA$$

$$= \iint_D \sqrt{4 + 25 + 1}\, dA$$

$$= \iint_D \sqrt{30}\, dA = \sqrt{30} \iint_D dA = \sqrt{30}\, A(D) = 9\sqrt{30}\,\pi$$

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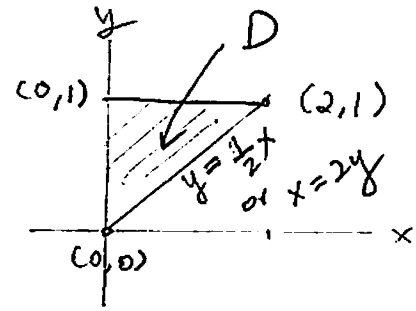
④ $z = 1 + 3x + 2y^2 = f(x,y)$

$f_x(x,y) = 3$

$f_y(x,y) = 4y$

$$\sqrt{(f_x)^2 + (f_y)^2 + 1} = \sqrt{16y^2 + 10}$$



$$A(S) = \iint_D \sqrt{(f_x)^2 + (f_y)^2 + 1}\, dA = \iint_D \sqrt{16y^2 + 10}\, dA$$

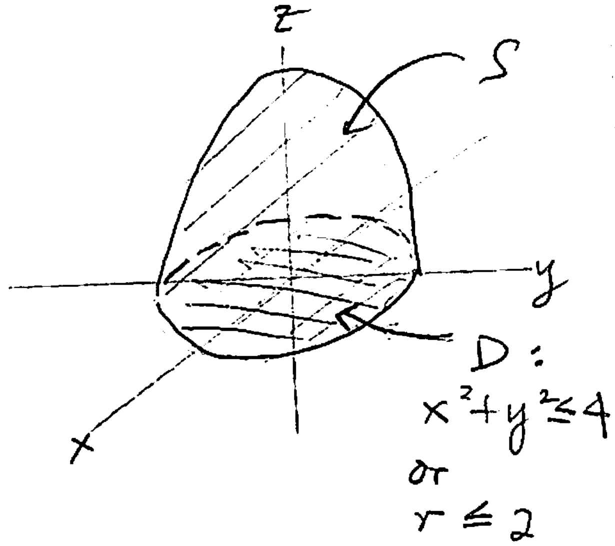
$$= \int_{y=0}^{1} \int_{x=0}^{2y} \sqrt{16y^2 + 10}\, dx\, dy$$

$$= \int_{y=0}^{1} \left[x\sqrt{16y^2 + 10} \right]_{x=0}^{2y} dy$$

Use substitution $u = 16y^2 + 10$

$$= \int_0^1 2y\sqrt{16y^2 + 10}\, dy = \frac{1}{24}\left(26^{3/2} - 10^{3/2}\right)$$

(6) The paraboloid $z = f(x,y) = 4 - x^2 - y^2$ intersects the xy-plane ($z=0$) when $x^2+y^2=4$.



$$f_x = -2x$$
$$f_y = -2y$$

$$\sqrt{(f_x)^2+(f_y)^2+1} = \sqrt{4x^2+4y^2+1} = \sqrt{4r^2+1}$$

$$A(S) = \iint_D \sqrt{(f_x)^2+(f_y)^2+1}\, dA = \int_{\theta=0}^{2\pi}\int_{r=0}^{2}\sqrt{4r^2+1}\; r\,dr\,d\theta$$
$$= \int_{\theta=0}^{2\pi} d\theta \int_{r=0}^{2} r\sqrt{4r^2+1}\, dr = 2\pi\left[\frac{1}{12}(4r^2+1)^{3/2}\right]_{r=0}^{2} = \frac{\pi}{6}\left(17\sqrt{17}-1\right)$$

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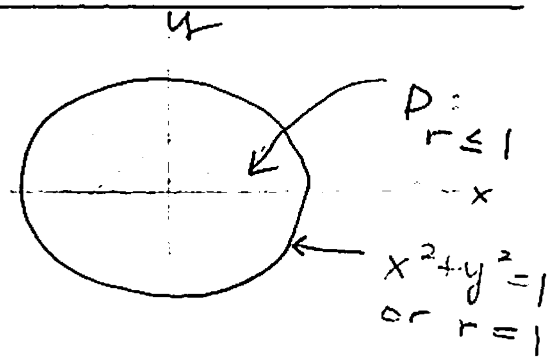
(9) $z = xy = f(x,y)$

$$f_x = y \qquad f_y = x$$

$$\sqrt{(f_x)^2+(f_y)^2+1} = \sqrt{x^2+y^2+1} = \sqrt{r^2+1}$$



$$A(S) = \iint_D \sqrt{(f_x)^2+(f_y)^2+1}\, dA = \int_{\theta=0}^{2\pi}\int_{r=0}^{1}\sqrt{r^2+1}\; r\,dr\,d\theta$$
$$= \int_{\theta=0}^{2\pi} d\theta \int_{r=0}^{1} r\sqrt{r^2+1}\, dr = \frac{2\pi}{3}\left(2\sqrt{2}-1\right)$$