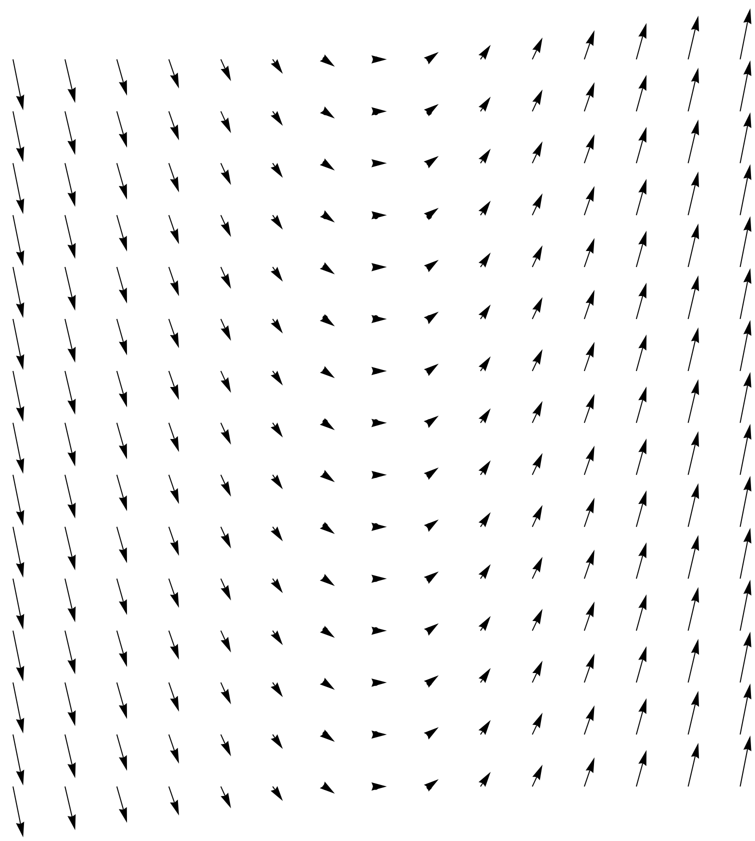
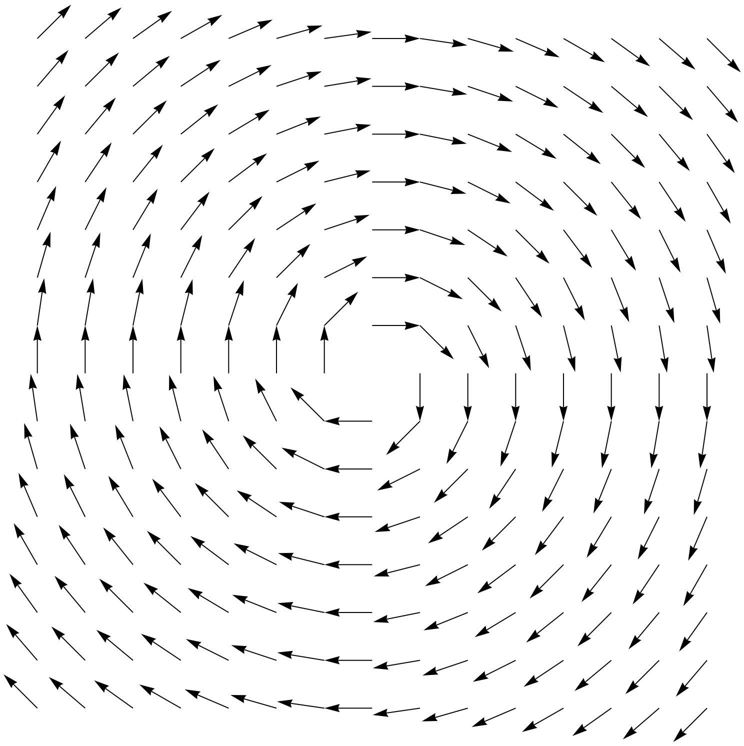
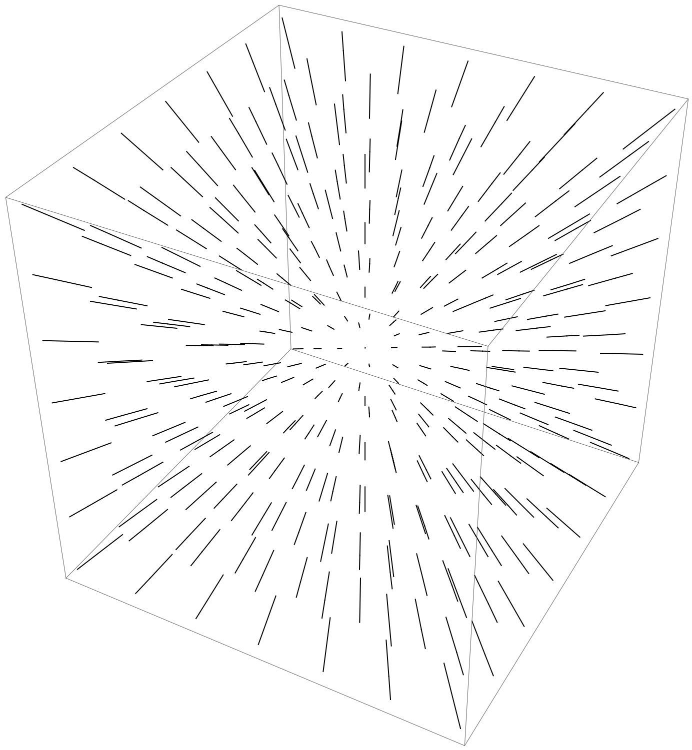
```
(* This Mathematica notebook exhibits plots of two-    and three-
 dimensional vector fields. The examples used are problems from section 16.1 of the text. *)

Needs["VectorFieldPlots`"]


(* Problem 2: We plot the vector field in the rectangle [-5,5]x[-5,5]. The
  first component of the field has a constant length of unity. Nevertheless,
the second component of the vector depends on the x-coordinates of the loci. Note
  that the field points downward where the x-coordinates are negative,
and points upward where the x-coordinates are positive. In particular,
the field is "horizontal" along the y-axis. *)

VectorFieldPlot[{1, x}, {x, -5, 5}, {y, -5, 5}]
```



```
(* Problem 6: We plot the vector field in the rectangle [-6,6]
  x[-6,6]. The vector field is NOT defined at the origin. Also,
it has a constant magnitude of unity. Furthermore, at each point (x,y) except the origin,
the field is orthogonal to the position vector <x,
y>. This can be verified by taking the dot product of <x,
y> with with the field. See also example 1 on p.1057 of the text. *)

VectorFieldPlot[{y / Sqrt[x^2 + y^2], -x / Sqrt[x^2 + y^2]}, {x, -6, 6}, {y, -6, 6}]
```



```
(* Problem 18: We plot the vector field in the box [-5,5]x[-5,5]x[-5,5]. *)
VectorFieldPlot3D[{x, y, z}, {x, -5, 5}, {y, -5, 5}, {z, -5, 5}]
```



Section 16.1 : Vector Fields

Problem 12 $\vec{F}(x,y) = \langle 1, \sin y \rangle$.

- The first component has a constant length of unity. In particular, it is positive. So the field always points to the right. There is only one such option, i.e. IV.
- In fact, note also that the second component depends sinusoidally on y: In particular,

$-\pi < y < 0$: $\longrightarrow \sin y > 0 \longrightarrow \vec{F}$ points downward ;

$0 < y < \pi$ $\longrightarrow \sin y < 0 \longrightarrow \vec{F}$ points upward ;

$\pi < y < 2\pi$ $\longrightarrow \sin y < 0 \longrightarrow \vec{F}$ points downward ; etc.

Problem 15 $\vec{F}(x,y,z) = \vec{i} + 2\vec{j} + 3\vec{k}$

This is a constant vector field. Clearly, the only option is IV.

Problem 18 $\vec{F}(x,y,z) = x\vec{i} + y\vec{j} + z\vec{k}$.

- The magnitude of the field depends only on the distance from the origin : $|\vec{F}(x,y,z)| = \sqrt{x^2+y^2+z^2}$. The only option is II.

Problem 23 $f(x,y,z) = \sqrt{x^2+y^2+z^2}$.

$$f_x(x,y,z) = \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$f_y(x,y,z) = \frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$f_z(x,y,z) = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\nabla f(x,y,z) = \frac{x\vec{i} + y\vec{j} + z\vec{k}}{\sqrt{x^2+y^2+z^2}}$$

Problem 31.

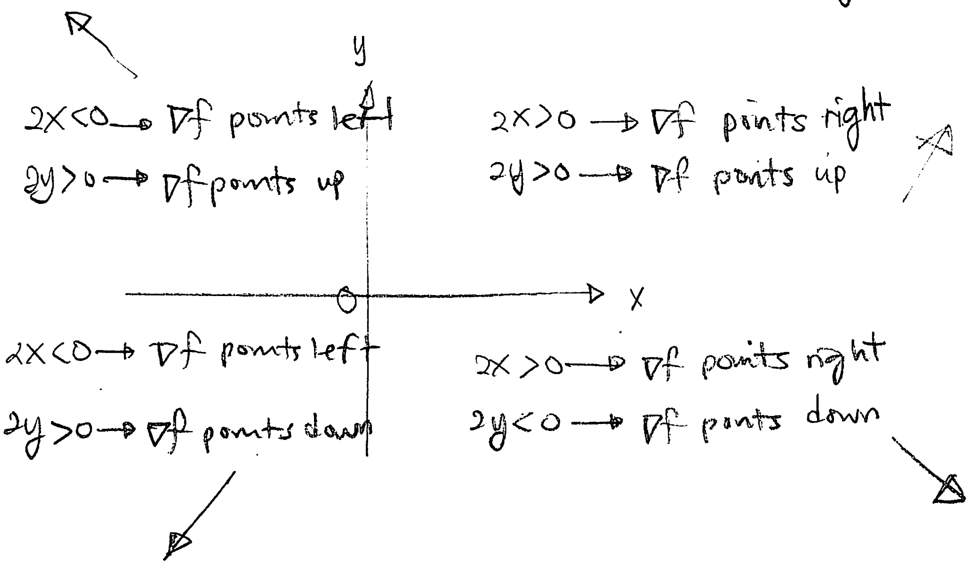
$f(x,y) = x^2 + y^2$

$f_x(x,y) = 2x$

$f_y(x,y) = 2y$

$\nabla f(x,y) = 2x\vec{i} + 2y\vec{j}$

$|\nabla f(x,y)| = 2\sqrt{x^2+y^2}$



There are two possible options: I and II.

But the magnitude of $\nabla f(x,y)$ changes with the distance of its locus (x,y) from the origin. This rules out I.

Thus, ∇f is given by II.