Homework 3

$\boxed{14.8.19}$ $f(x,y) = e^{-xy}$, $\underbrace{x^2+4y^2}_{g(x,y)} \le 1$

Step 1: Find critical point(s) of f in the interior of $x^2+4y^2 \le 1$. $(x^2+4y^2<1)$

1 $f_x = -ye^{-xy} = 0$
1 $f_y = -xe^{-xy} = 0$ $\}$ 1 $(0,0)$ is the only critical point

Step 2: Find candidates of extreme values on the boundary of $x^2+4y^2 \le 1$ $(x^2+4y^2=1)$

(1) $-ye^{-xy} = f_x = \lambda g_x = 2\lambda x$ 1
(2) $-xe^{-xy} = f_y = \lambda g_y = 8\lambda y$ 1
(3) $x^2+4y^2=1$

1 Note that from (1) and (2), if $x=0$ then $y=0$ and if $y=0$ then $x=0$.
But (3) forbids that $x=0=y$. So we can assume $x \neq 0$, $y \neq 0$.

Now

x times (1) $\rightarrow -xye^{-xy} = 2\lambda x^2$
y times (2) $\rightarrow -xye^{-xy} = 8\lambda y^2$ $\}$ $2\lambda x^2 = 8\lambda y^2$

$\downarrow$ $\lambda \neq 0$, otherwise $x=0=y$.

$x^2 = 4y^2$

Hence (3) $\rightarrow$ $2x^2 = 8y^2 = 1$

$\rightarrow \begin{cases} x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2} & 2 \\ y = \pm\frac{1}{\sqrt{8}} = \pm\frac{1}{2\sqrt{2}} = \pm\frac{\sqrt{2}}{4} & 2 \end{cases}$

Step 3: List all values of f at points found in steps 1 & 2

$f(0,0) = 1$ 1
$f\left(\pm\frac{\sqrt{2}}{2}, \pm\frac{\sqrt{2}}{4}\right) = e^{-\frac{1}{4}}$ 1 $\simeq$ ~~0.882497~~ 0.77880078[3] minimum 1
$f\left(\pm\frac{\sqrt{2}}{2}, \mp\frac{\sqrt{2}}{4}\right) = e^{+\frac{1}{4}}$ 1 maximum. 1

~~1.13315~~
1.1284025417