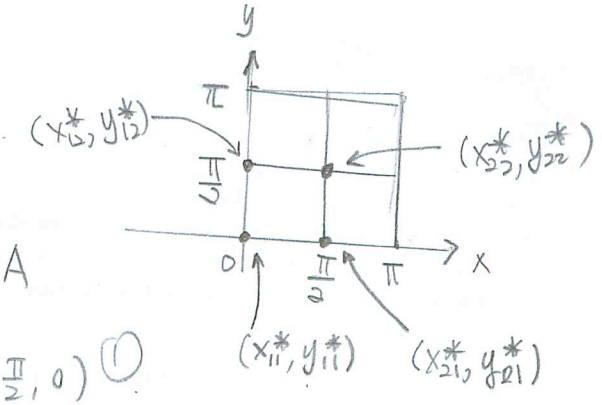
Homework #4

15.1.3(a)

$\Delta A = \Delta x \Delta y = \frac{\pi^2}{4}$ ①



$$\iint_R \underbrace{\sin(x+y)}_{f(x,y)}\, dA \simeq \sum_{i=1}^{2}\sum_{j=1}^{2} f(x_{ij}^*, y_{ij}^*)\,\Delta A$$

$$= \left[f(0,0) + f\left(0,\tfrac{\pi}{2}\right) + f\left(\tfrac{\pi}{2},0\right) + f\left(\tfrac{\pi}{2},\tfrac{\pi}{2}\right)\right]\Delta A$$

① ① ① ① ①

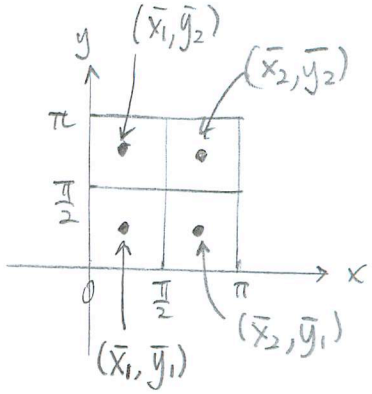
$$= \left(0 + 1 + 1 + 0\right)\left(\frac{\pi^2}{4}\right)$$

① + ①

$$= \frac{\pi^2}{2} \simeq 4.935$$

①

15.1.3(b)



$$\iint_R \sin(x+y)\, dA \simeq \sum_{i=1}^{2}\sum_{j=1}^{2} f(\bar{x}_i, \bar{y}_j)\,\Delta A$$

① ①

$$= \left[f\left(\tfrac{\pi}{4},\tfrac{\pi}{4}\right) + f\left(\tfrac{\pi}{4},\tfrac{3\pi}{4}\right) + f\left(\tfrac{3\pi}{4},\tfrac{\pi}{4}\right) + f\left(\tfrac{3\pi}{4},\tfrac{3\pi}{4}\right)\right]\Delta A$$

① ① ① ①

① + ①

$$= \left[1 + 0 + 0 + (-1)\right]\left(\frac{\pi^2}{4}\right)$$

$$= 0$$ ①

Exact value of the integral is

$$\iint_R \sin(x+y)\,dA = \int_0^{\pi}\int_0^{\pi} \sin(x+y)\,dx\,dy = \cdots = 0$$

* Note that $\sin(x+y)$ is positive somewhere and negative somewhere else on R. So it is illegitimate to interpret $\iint_R \sin(x+y)\,dA$ as a volume.