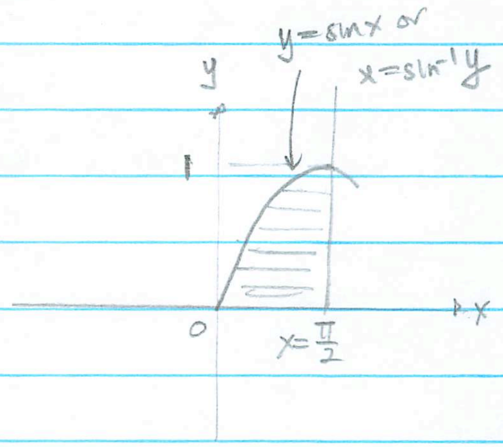
HW #5

15.3.47



$$\int_0^1 \int_{\sin^{-1} y}^{\frac{\pi}{2}} \cos x \sqrt{1+\cos^2 x}\, dx\, dy$$

$$= \int_0^{\frac{\pi}{2}} \int_0^{\sin x} \cos x \sqrt{1+\cos^2 x}\, dy\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sin x \cos x \sqrt{1+\cos^2 x}\, dx$$

suggested by a student → $u = 1+\cos^2 x$

$$= -\int_2^1 \sqrt{u}\, \frac{du}{2}$$

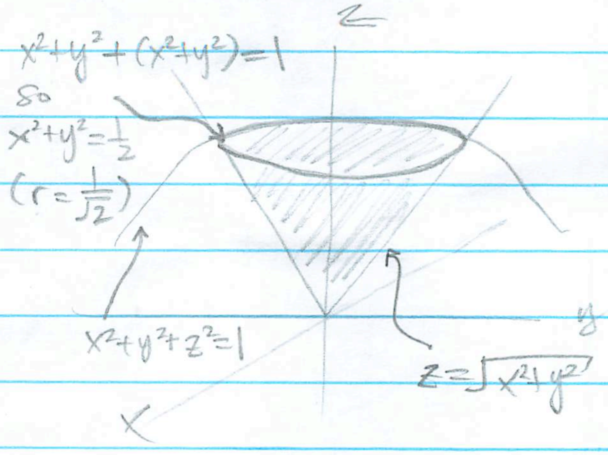
$-\frac{du}{2} = \sin x \cos x\, dx$

one can also use the substitution $u = \cos x$.

$$= -\left[\frac{u^{\frac{3}{2}}}{3}\right]_2^1 = \frac{1}{3}(2\sqrt{2}-1)$$

≈ 0.609475

15.4.25



$$V = \iint_{x^2+y^2 \le \frac{1}{2}} \left(\sqrt{1-x^2-y^2} - \sqrt{x^2+y^2}\right) dA$$

$$= \int_0^{2\pi} \int_0^{\frac{1}{\sqrt{2}}} \left(\sqrt{1-r^2} - r\right) r\, dr\, d\theta$$

$$= \int_0^{2\pi} d\theta \int_0^{\frac{1}{\sqrt{2}}} \left(r\sqrt{1-r^2} - r^2\right) dr$$

$$= 2\pi \left[-\frac{1}{3}(1-r^2)^{3/2} - \frac{r^3}{3}\right]_0^{\frac{1}{\sqrt{2}}}$$

$$= \frac{\pi}{3}(2-\sqrt{2})$$

$\approx 0.61343...$