Math 263A - Quiz #1 – Wi0~~8~~9 (OSU)

January 15, 200~~8~~9

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Name:

Signature:

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Instructions. *The point value of each problem is indicated in square brackets. To obtain full credit, you must have the correct answers along with relevant supporting work to justify them. Partial credit will be given based on the work that is shown. However,* ***answers without supporting work will receive no credit.***

~~Problem. Test the following series for convergence or divergence. You must name the tests you use and justify the hypotheses in the tests as we have done in class.~~

(a) Use the squeeze theorem to show that $\lim_{(x,y)\to(0,0)} \frac{x^2\sin^2 y}{x^2+y^2}=0$. [7pts]

$$0 \le \frac{x^2\sin^2 y}{x^2+y^2} = \frac{x^2}{x^2+y^2}\sin^2 y \le \sin^2 y$$

as $(x,y)\to(0,0)$ (2): $0 \to 0$ (1)

(3) as $(x,y)\to(0,0)$: $\sin^2 y \to 0$ (1)

$$\text{So } \lim_{(x,y)\to(0,0)} \frac{x^2\sin^2 y}{x^2+y^2} = 0$$

Along $y=x$, $\frac{x^4y}{x^8+y^2} = \frac{x^5}{x^8+x^2} = \frac{x^3}{x^6+1} \to 0$ as $(x,y)\to(0,0)$

(b) Show that the limit $\lim_{(x,y)\to(0,0)} \frac{x^4 y}{x^8+y^2}$ does not exist. [8pts]

(2) Along $x=0$, $\frac{x^4y}{x^8+y^2} \overset{y\neq 0}{=} 0 \longrightarrow 0$ as $(x,y)\to(0,0)$ (1)

(or along $y=0$, $\frac{x^4y}{x^8+y^2} \overset{x\neq 0}{=} 0 \longrightarrow 0$ as $(x,y)\to(0,0)$)

(2) Along $y=x^4$, $\frac{x^4y}{x^8+y^2} \overset{x\neq 0}{=} \frac{x^4x^4}{x^8+x^8} = \frac{1}{2} \longrightarrow \frac{1}{2}$ as $(x,y)\to(0,0)$.

Since $0 \neq \frac{1}{2}$, $\lim_{(x,y)\to(0,0)} \frac{x^4y}{x^8+y^2}$ does not exist.

(1)