

Instructions. The point value of each problem is indicated in square brackets. To obtain full credit, you must have the correct answers along with relevant supporting work to justify them. Partial credit will be given based on the work that is shown. However, **answers without supporting work will receive no credit.**

Consider the surface $F(x, y, z) = 0$, where $F(x, y, z) = yz - \ln(x+z)$, for the following problems.

1. Find the gradient $\nabla F(0, 0, 1)$. [5pts]

$$\begin{aligned} \textcircled{1} \quad F_x &= -\frac{1}{x+z} \frac{(0,0,1)}{1} = -1 \\ \textcircled{1} \quad F_y &= z \frac{(0,0,1)}{1} = 1 \\ \textcircled{1} \quad F_z &= y - \frac{1}{x+z} \frac{(0,0,1)}{1} = -1 \end{aligned} \quad \nabla F(0,0,1) = \langle -1, 1, -1 \rangle$$

2. Find an equation of the tangent plane to the surface at the point $(0, 0, 1)$. Simplify the equation as far as possible. [2pts]

$$\begin{aligned} \textcircled{1} \quad \nabla F(0,0,1) \cdot \langle x-0, y-0, z-1 \rangle &= 0 \\ -x + y - z + 1 &= 0 \\ \textcircled{1} \quad x - y + z &= 1 \end{aligned}$$

3. Find the parametric equations of the normal line to the surface at the point $(0, 0, 1)$. [5pts]

$$\begin{aligned} x &= 0 + (-1)t = -t && \textcircled{1} + \textcircled{1} \\ y &= 0 + (1)t = t && \textcircled{1} + \textcircled{1} \\ z &= 1 + (-1)t = 1-t && \textcircled{1} \end{aligned}$$

4. Calculate the directional derivative $D_{\vec{u}}F(0, 0, 1)$, where \vec{u} is a unit vector in the direction of the vector $\langle 1, 1, 1 \rangle$. [3pts]

$$\begin{aligned} \vec{u} &= \frac{\langle 1, 1, 1 \rangle}{|\langle 1, 1, 1 \rangle|} = \frac{1}{\sqrt{3}} \langle 1, 1, 1 \rangle && \textcircled{1} \\ D_{\vec{u}}F(0,0,1) &= \nabla F(0,0,1) \cdot \frac{1}{\sqrt{3}} \langle 1, 1, 1 \rangle && \textcircled{1} \\ &= \frac{-1+1-1}{\sqrt{3}} = -\frac{1}{\sqrt{3}} && \textcircled{1} \end{aligned}$$