Math 263A - Quiz # 6 - Wi09(OSU)
March 5, 2009

Name:________________
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Instructions. The point value of each problem is indicated in square brackets. To obtain full credit, you must have the correct answers along with relevant supporting work to justify them. Partial credit will be given based on the work that is shown. However, **answers without supporting work will receive no credit.**

Problem 1. Supposing that the vector field $\mathbf{F}(x,y) = y\mathbf{i} + (x+2y)\mathbf{j}$ is conservative, find a function f such that $\nabla f = \mathbf{F}$. Then compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ for all smooth curves C in $\mathbb{R}^2$ that begins at $(0,1)$ and ends at $(2,1)$. [8pts]

① $f_x = y$

② $f_y = x + 2y$

① $\rightarrow$ $f(x,y) = \int y\, dx$

$= xy + g(y)$

③ $f_y = x + g'(y)$

②,③ $\rightarrow$ $g'(y) = 2y$

$g(y) = y^2 + K$

So $f(x,y) = xy + y^2 + K$

Alternatively

② $\rightarrow$ $f(x,y) = \int (x+2y)\, dy$

$= xy + y^2 + h(x)$

③' $f_x = y + h'(x)$

①,③' $\rightarrow$ $h'(x) = 0$

$h(x) = K$

So $f(x,y) = xy + y^2 + K$

$$\int_C \vec{F}\cdot d\vec{r} = \int_C \nabla f \cdot d\vec{r}$$
$$= f(2,1) - f(0,1)$$
$$= (2+1) - (0+1)$$
$$= 2$$

Problem 2. Use Green's theorem to calculate the line integral $\int_C e^{x^4}\sin(x^2)\, dx + \cos(\sin(y))\, dy$, where C is the ellipse $4x^2 + 9y^2 = 1$, positively oriented. Show all your work. [7pts]

$\frac{\partial}{\partial x}\cos(\sin(y)) = 0$

$\frac{\partial}{\partial y} e^{x^4}\sin(x^2) = 0$

$$\int_C e^{x^4}\sin(x^2)\, dx + \cos(\sin(y))\, dy$$
$$= \iint_{4x^2+9y^2\le 1} \frac{\partial}{\partial x}\cos(\sin(y)) - \frac{\partial}{\partial y} e^{x^4}\sin(x^2)\, dA$$
$$= \iint_{4x^4+9y^2\le 1} 0\, dA$$
$$= 0$$