Math 263A - Quiz # 7 - Wi09(OSU)
March 12, 2009

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Instructions. The point value of each problem is indicated in square brackets. To obtain full credit, you must have the correct answers along with relevant supporting work to justify them. Partial credit will be given based on the work that is shown. However, **answers without supporting work will receive no credit.**

Let S be the part of the surface $z = xe^y$ above the square region $D = \{(x, y, z) | 0 \leq x \leq 1, 0 \leq y \leq 1, z = 0\}$ and has the downward orientation.

Problem 1. Give a vector function $\mathbf{r}(x, y)$ that parametrizes S. [2pts]

$$\left.\begin{array}{l} x = x \\ y = y \\ z = xe^y \end{array}\right\} \vec{r}(x,y) = \langle x, y, xe^y \rangle \text{ with domain}$$

$$D: 0 \leq x \leq 1, \quad 0 \leq y \leq 1$$

Problem 2. Calculate $\mathbf{r}_x \times \mathbf{r}_y$. [3pts]

points upward

$$\vec{r}_x = \langle 1, 0, e^y \rangle$$

$$\vec{r}_y = \langle 0, 1, xe^y \rangle$$

$$\vec{r}_x \times \vec{r}_y = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & e^y \\ 0 & 1 & xe^y \end{vmatrix} = \langle -e^y, -xe^y, 1 \rangle$$

Problem 3. Write an interated integral that gives $A(S)$, the surface area of S. Do NOT evaluate the integral. [3pts]

$$A(S) = \iint_S 1 \, dS$$

$$= \iint_D |\vec{r}_x \times \vec{r}_y| \, dA$$

$$= \iint_D \sqrt{1 + e^{2y} + x^2 e^{2y}} \, dA$$

$$= \int_0^1 \int_0^1 \sqrt{1 + e^{2y} + x^2 e^{2y}} \, dx \, dy$$

Problem 4. Compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = xy\mathbf{i} + 4x^2\mathbf{j} + yz\mathbf{k}$. [7pts]

$$\iint_S \vec{F} \cdot d\vec{S}$$

points downward

$$= \iint_D \vec{F}(\vec{r}(x,y)) \cdot (-\vec{r}_x \times \vec{r}_y) \, dA$$

$$= \iint_D \langle xy, 4x^2, xye^y \rangle \cdot \langle e^y, xe^y, -1 \rangle \, dA$$

$$= \iint_D 4x^3 e^y \, dA$$

$$= \int_0^1 \int_0^1 4x^3 e^y \, dx \, dy$$

$$= \int_0^1 4x^3 \, dx \int_0^1 e^y \, dy$$

$$= [x^4]_0^1 [e^y]_0^1$$

$$= e - 1$$