Math 415A: 1.3 : Classification of DEs

1

$$t^2\frac{d^2y}{dt^2}+t\frac{dy}{dt}+2y=\sin t$$

order $=2$; linear because it has the form

$$\underbrace{a_0(t)}_{t^2}y''+\underbrace{a_1(t)}_{t}y'+\underbrace{a_2(t)}_{2}y=\underbrace{g(t)}_{\sin t}$$

2

$$(1+y^2)\frac{d^2y}{dt^2}+t\frac{dy}{dt}+y=e^t$$

order $=2$; nonlinear

this is the term that makes it nonlinear

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$$\frac{d^4y}{dt^4}+\frac{d^3y}{dt^3}+\frac{d^2y}{dt^2}+\frac{dy}{dt}+y=1$$

order $=$? It is linear if it is of the form

$a_0(t)y^{(4)}+a_1(t)y^{(3)}+a_2(t)y^{(2)}+a_3(t)y'+a_4(t)y=g(t)$.

What are the functions $a_0(t)$? $a_1(t)$? $a_2(t)$? $a_3(t)$? $a_4(t)$? and $g(t)$ in this case?

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$$\frac{dy}{dt}+ty^2=0$$

order $=$? linear or nonlinear?

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$$y''''+4y'''+3y=t \quad (*)$$

$y_1=\frac{t}{3}$ $\quad y_1''=y_1'''=y_1''''=0$

$y_1'=\frac{1}{3}$

So $y_1''''+4y_1'''+3y_1=0+0+3\left(\frac{t}{3}\right)$

$=t$,

i.e., y_1 satisfies $(*)$ and it is therefore a solution of $(*)$.

$y_2=e^{-t}+\frac{t}{3}$

$y_2'=-e^{-t}+\frac{1}{3}$

$y_2''=e^{-t}$

$y_2'''=-e^{-t}$

$y_2''''=e^{-t}$

So $y_2''''+4y_2'''+3y_2$

$=e^{-t}+4(-e^{-t})+3\left(e^{-t}+\frac{t}{3}\right)$

$=t$, i.e.,

y_2 is also a solution.

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$$2t^2y''+3ty'-y=0,\quad t>0 \quad (*)$$

$$\left.\begin{aligned} y_1&=t^{1/2}\\ y_1'&=\tfrac{1}{2}t^{-1/2}\\ y_1''&=-\tfrac{1}{4}t^{-3/2}\end{aligned}\right\}\quad \begin{aligned}&2t^2y_1''+3ty_1'-y\\ &=2t^2\left(-\tfrac{1}{4}t^{-3/2}\right)+3t\left(\tfrac{1}{2}t^{-1/2}\right)-t^{1/2}\\ &=-\tfrac{1}{2}t^{1/2}+\tfrac{3}{2}t^{1/2}-t^{1/2}\\ &=0\end{aligned}$$

so y_1 is a solution of (*)

Do the same for $y_2 = \frac{1}{t}$

12 $t^2y'' + 5ty' + 4y = 0, \; t > 0$ (*)

$y_1 = t^{-2}, \; y_1' = -2t^{-3}, \; y_1'' = 6t^{-4}$

So

$$t^2 y_1'' + 5t y_1' + 4y_1$$
$$= t^2(6t^{-4}) + 5t(-2t^{-3}) + 4t^{-2}$$
$$= 6t^{-2} - 10t^{-2} + 4t^{-2}$$
$$= 0,$$

so that y_1 is a solution of (*)

Also, $y_2 = t^{-2}\ln t$

$$y_2' = -2t^{-3}\ln t + t^{-3}$$
$$y_2'' = 6t^{-4}\ln t - 2t^{-4} - 3t^{-4}$$
$$= 6t^{-4}\ln t - 5t^{-4}$$

Hence

$$t^2 y_2'' + 5t y_2' + 4y_2$$
$$= t^2(6t^{-4}\ln t - 5t^{-4})$$
$$+ 5t(-2t^{-3}\ln t + t^{-3})$$
$$+ 4t^{-2}\ln t$$
$$= 6t^{-2}\ln t - 5t^{-2}$$
$$- 10t^{-2}\ln t + 5t^{-2}$$
$$+ 4t^{-2}\ln t$$

$= 0$, so y_2 is also a solution of (*)