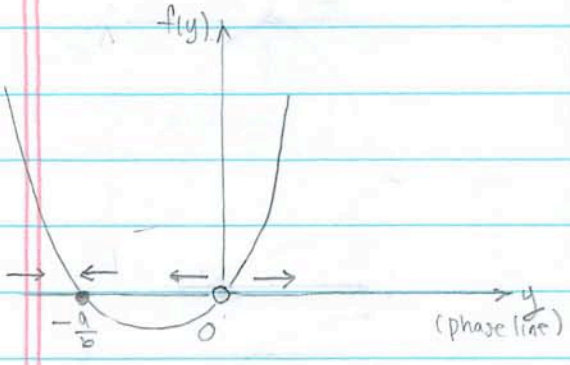
Math 415A Sec 2.5: Autonomous Equations and Population Dynamics

2 $\frac{dy}{dt} = \underbrace{ay+by^2}_{f(y)}$, $a>0, b>0$, $-\infty < y_0 < \infty$



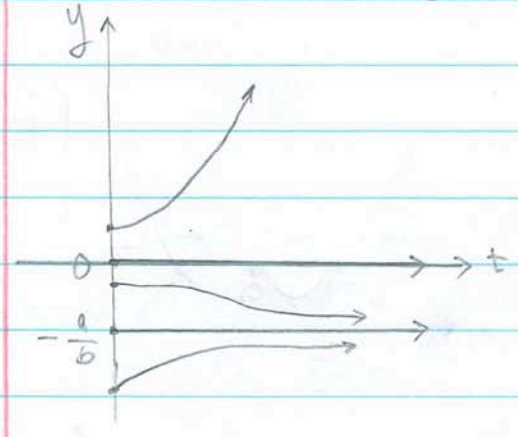
$f(y) = ay + by^2$

$= y(a+by)$

$= 0$ only if

Eq. solutions: $y=0$ or $y = -\frac{a}{b}$

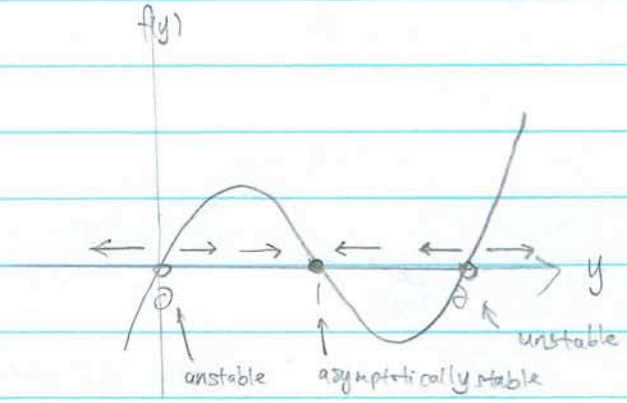
$y=0$: unstable; $y = -\frac{a}{b}$: asymptotically stable

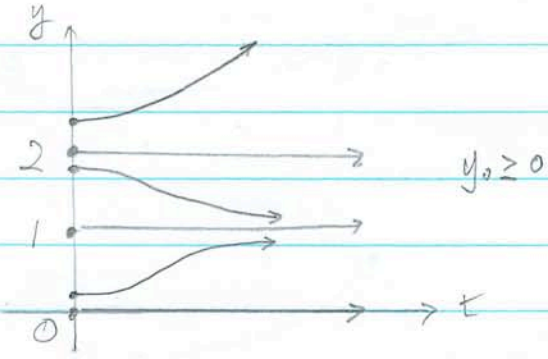


*3 $\frac{dy}{dt} = \underbrace{y(y-1)(y-2)}_{f(y)}$, $y_0 \geq 0$

$f(y) = 0$ only if

Eq. solutions: $y=0$, $y=1$, or $y=2$

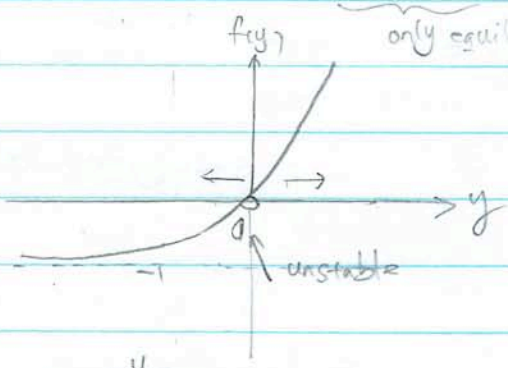


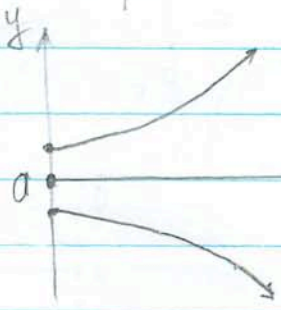


4 $\frac{dy}{dt} = \underbrace{e^y - 1}_{f(y)}$, $-\infty < y_0 < \infty$

$f(y) = 0$ only if $e^y - 1 = 0$

or $e^y = 1$ or $y = \ln 1 = 0$ (only equilibrium solution)





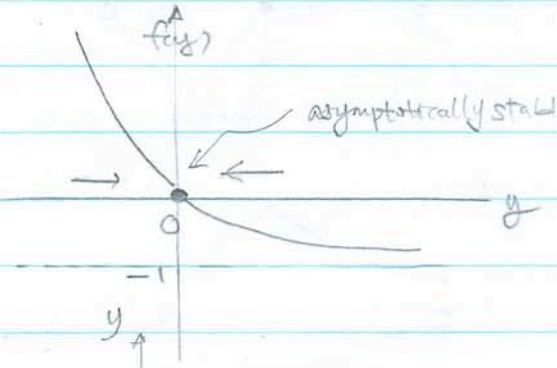
*5 $\frac{dy}{dt} = \underbrace{e^{-y} - 1}_{f(y)}, \quad -\infty < y_0 < \infty$

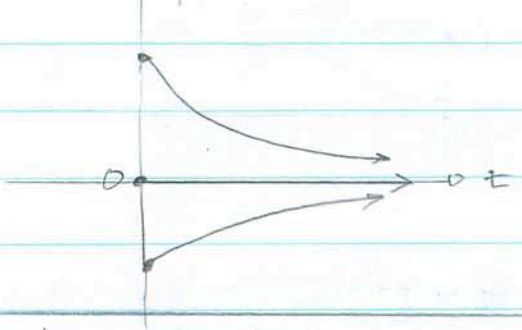
$f(y) = 0$ only if

$e^{-y} = 1$

or $-y = \ln 1 = 0$

or $y = 0$ ← only equilibrium solution



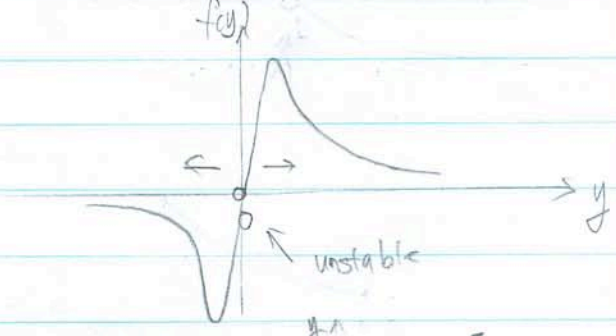


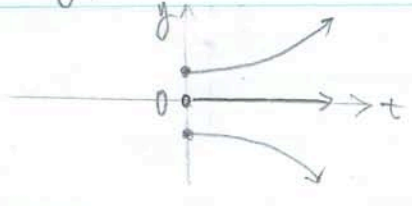
6 $\frac{dy}{dt} = \underbrace{\frac{-2 \arctan y}{1+y^2}}_{f(y)}, \quad -\infty < y_0 < \infty$

$f(y) = 0$ only if

$\arctan y = 0$

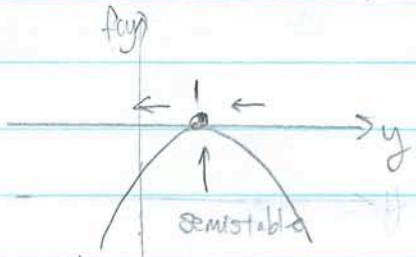
or $y = 0$ ← only equilibrium solution

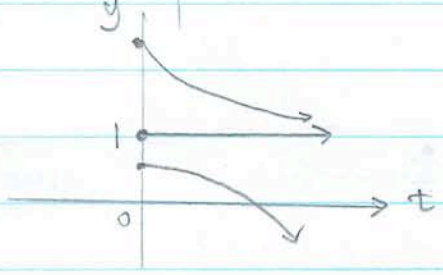




8 $\frac{dy}{dt} = \underbrace{-k(y-1)^2}_{f(y)}, \quad k > 0, \quad -\infty < y_0 < \infty$

$f(y) = 0$ only if $y = 1$ ← only eq. solution.

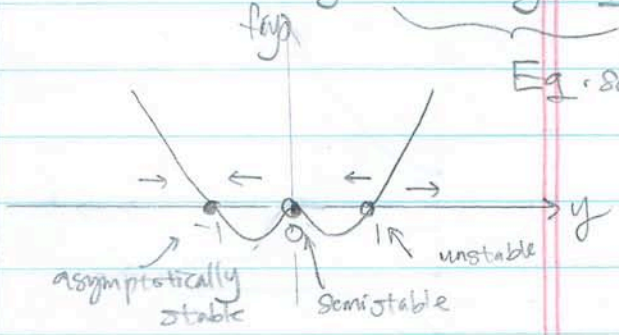


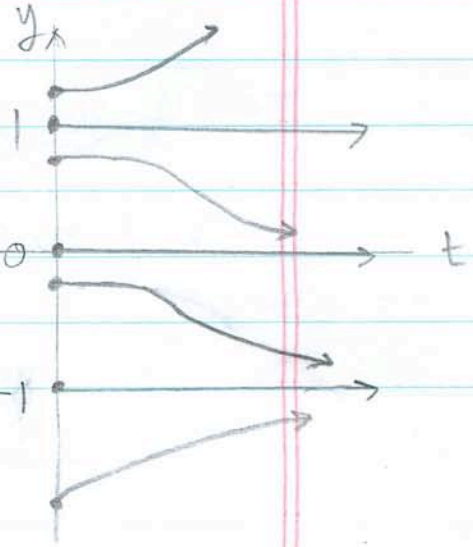


9 $\frac{dy}{dt} = \underbrace{y^2(y^2-1)}_{f(y)}, \quad -\infty < y_0 < \infty$

$f(y) = 0$ only if $\underbrace{y^2 = 0}_{y=0}$ or $\underbrace{y^2 - 1 = 0}_{y = \pm 1}$

Eq. solutions





*10 $\frac{dy}{dt} = y(1-y^2)$, $-\infty < y_0 < \infty$

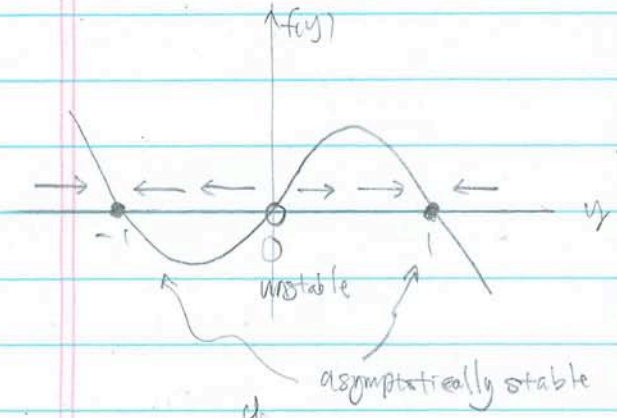
$f(y)$

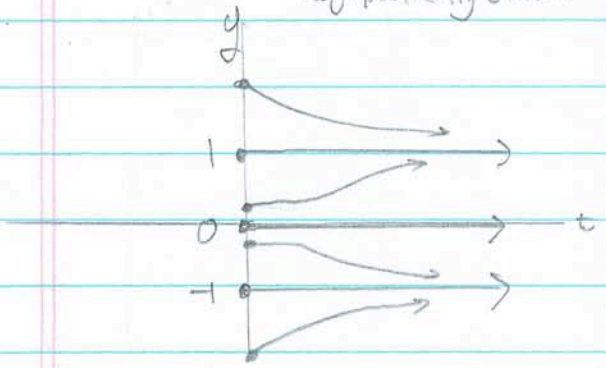
$f(y) = 0$ only if

$y = 0$ or $1 - y^2 = 0$

$y = \pm 1$

eq. sol's





11 $\frac{dy}{dt} = ay - b\sqrt{y}$, $a > 0$, $b > 0$, $y_0 \geq 0$

$f(y)$

$f(y) = 0$ only if

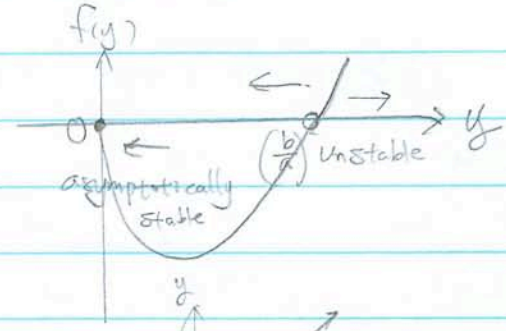
$ay - b\sqrt{y} = 0$

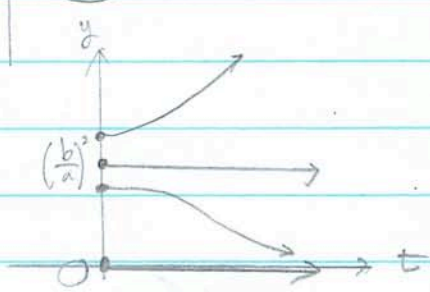
$\sqrt{y}(a\sqrt{y} - b) = 0$

$\sqrt{y} = 0$ or $\sqrt{y} = \frac{b}{a}$

$y = 0$ $y = \left(\frac{b}{a}\right)^2$

eq. sol's





12 $\frac{dy}{dt} = y^2(4-y^2)$, $-\infty < y_0 < \infty$

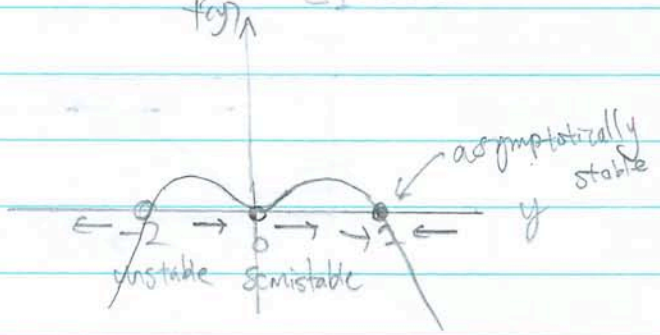
$f(y)$

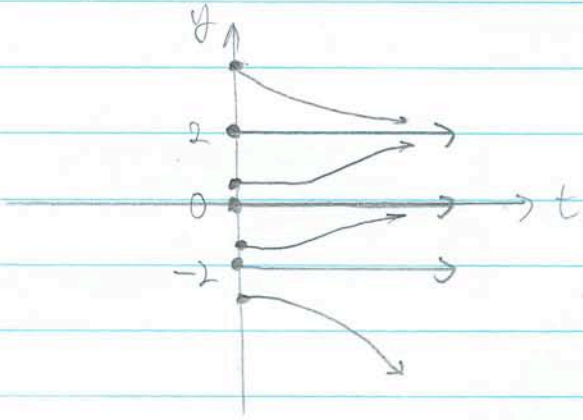
$f(y) = 0$ only if

$y^2 = 0$ or $4 - y^2 = 0$

$y = 0$ or $y = \pm 2$

eq. sol's





*13 $\frac{dy}{dt} = \underbrace{y^2(1-y)^2}_{f(y)}, \quad -\infty < y_0 < \infty$

$f(y) = 0$ only if

$y^2 = 0$ or $(1-y)^2 = 0$

$y = 0$ or $1-y = 0$

$y = 1$

eq-soln's