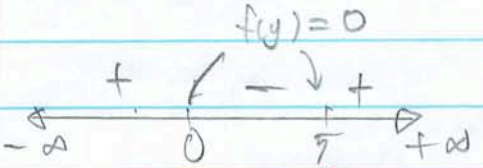
1.1: 4, 12, 22-25
1.2: 8, 13

Math 417A Graded Homework #1

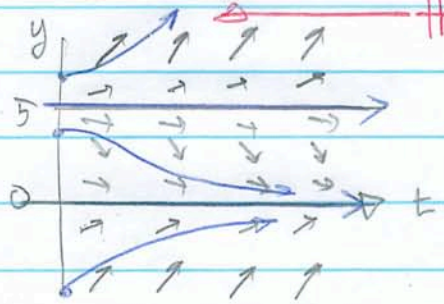
(2) 1.1.12 $y' = -y(5-y) \equiv f(y)$

Step 1: $-y(5-y) = 0 \rightarrow y = 0$ or $y = 5$ equilibrium solutions

Step 2:



Step 3:



This is not well-defined for all $t \geq 0$. It diverges to $+\infty$ in finite time

NOTE: the direction field is nowhere vertical, since otherwise $y' = +\infty$. But $y' = -y(5-y)$. So y' is finite for any finite value of y!

If $y(0) < 5$, $\lim_{t\to\infty} y(t) = 0$

If $y(0) = 5$, $\lim_{t\to\infty} y(t) = 5$

If $y(0) > 5$, $y(t)$ diverges to $+\infty$ as t approaches some finite $\bar{t}$.

(3) 1.2.8 $\frac{dp}{dt} = rp, \quad r > 0$

$$\int \frac{dp}{p} = \int r\,dt$$

$$p(t) = Ae^{rt}$$

But $p(0) = Ae^{r(0)}$, so $A = p(0)$ and

$$p(t) = p(0)e^{rt}$$

(a) $p(0)e^{r(30\text{ days})} = p(30\text{ days}) = 2p(0)$

So $r = \frac{\ln 2}{30}\,\text{day}^{-1} \simeq 0.231\,\text{day}^{-1}$

(b) Likewise, $p(0)\,e^{r(N\text{ days})} = p(N\text{ days}) = 2p(0)$

So $r = \frac{0.2}{N}\,\text{day}^{-1}$