3.2: 9, 12, 17, 24, 25
3.3: 3, 12, 15
3.4: 11, 22

Math 415 A: Homework #4

3.2.17 $W(f,g) = 3e^{4t}$, $f(t) = e^{2t}$ ← given.

$$fg' - gf' = 3e^{4t} \quad \tfrac{1}{2}$$
$$e^{2t}g' - 2e^{2t}g = 3e^{4t}$$
$$g' - 2g = 3e^{2t} \quad \tfrac{1}{2}$$

$$\mu(t) = \exp\int -2\,dt = e^{-2t} \quad \tfrac{1}{2}$$

$$e^{-2t}g(t) = \int 3e^{2t}e^{-2t}\,dt = \int 3\,dt \quad \tfrac{1}{2}$$
$$g(t) = 3te^{2t} + Ce^{2t} \quad \tfrac{1}{2}$$

Second part of **3.2.24** $y'' - 2y' + y = 0$, $y_1 = e^t$, $y_2 = te^t$

Need to check that y_1 and y_2 are indeed solutions in the first part!!

$$W(y_1, y_2)(t) = y_1y_2' - y_2y_1' \quad \tfrac{1}{2}$$
$$= e^t(te^t + e^t) - te^t(e^t) \quad \tfrac{1}{2} \quad \tfrac{1}{2}$$
$$= e^{2t}$$
$$\neq 0. \quad \tfrac{1}{2}$$

Thus y_1 and y_2 are linearly independent.
So they form a fundamental set of solutions

Alternatively Say

$$(*) \quad k_1y_1(t) + k_2y_2(t) = 0 \quad \text{for all } t$$
$$k_1e^t + k_2te^t = 0 \quad \text{for all } t$$
$$e^t(k_1 + k_2t) = 0 \quad \text{for all } t$$
$$k_1 + k_2t = 0 \quad \text{for all } t \text{ (since } e^t > 0)$$

In particular, if $t = 0$, $k_1 = 0$.
Thus $k_2t = 0$ for all t
In particular, if $t = 1$, $k_2 = 0$.
Hence $(*)$ holds only if $k_1 = k_2 = 0$. So y_1 and y_2 are linearly independent