

Math 415A: Homework #5

3.5: 3, 4, 14, 23, 27

3.6: 2, 5, 9, 15, 18

3.8: 6, 7, 8, 11

3.5.23 $t^2 y'' - 4ty' + 6y = 0, t > 0, y_1 = t^2$

$y_2 = y_1 z = t^2 z$ $\left(\frac{1}{2}\right)$

$y_2' = t^2 z' + 2tz$

$y_2'' = t^2 z'' + 4tz' + 2z$

$$\left. \begin{array}{l} t^2 y_2'' = t^4 z'' + 4t^3 z' + 2t^2 z \\ -4ty_2' = -4t^3 z' - 8t^2 z \\ 6y = 6t^2 z \end{array} \right\} \oplus = t^4 z'' = 0 \quad \left(\frac{1}{2}\right)$$

So $z'' = 0$ $\left(\frac{1}{2}\right)$

$z = C_1 t + C_2, C_1 = 1, C_2 = 0$

$y_2 = C_1 t^3 + C_2 t^2 = t^3$ $\left(\frac{1}{2}\right)$

3.6.15 $y'' - 2y' + y = te^t + 4, y(0) = 1, y'(0) = 1$

Step 1: $y'' - 2y' + y = 0 \rightarrow y_1 = e^t, y_2 = te^t$
 $(r-1)^2 = 0$ $\left(\frac{1}{2}\right)$

Step 2: $Y = t^2(At+B)e^t + C$ $\left(\frac{1}{2}\right) \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)$
 $t^2 y_1 z'' + (2t^2 y_1' - 4ty_1) z' = 0$

$$\left. \begin{array}{l} Y'' - 2Y' + Y = te^t + 4 \\ 6At e^t + 2B e^t + C \end{array} \right\} \begin{array}{l} A = \frac{1}{6} \\ B = 0 \\ C = 4 \end{array} \quad \left(\frac{1}{2}\right)$$

$Y = \frac{1}{6} t^3 e^t + 4$

$y = c_1 e^t + c_2 t e^t + \frac{1}{6} t^3 e^t + 4$

$y(0) = 1, y'(0) = 1: c_1 = -3, c_2 = 4$

$y = -3e^t + 4te^t + \frac{1}{6} t^3 e^t + 4$ $\left(\frac{1}{2}\right)$