HW#7:
10.3: 1, 4, 14, 15, 16
10.4: 7, 15, 17
10.5: 1, 2, 8, 9

average values of one-sided limits

So the value of the Fourier series at these points are the corresponding

10.3.1

$$f(x) = \begin{cases} -1 & (-1 \le x < 0) \\ +1 & (0 \le x < 1) \end{cases}$$

odd × even = odd

$$a_n = \frac{1}{1} \int_{-1}^{1} f(x) \cos \frac{n\pi x}{1}\, dx = 0 \quad \text{for } n = 0, 1, 2, \dots \quad \tfrac{1}{2}$$

half the period

odd × odd = even

$$b_n = \frac{1}{1} \int_{-1}^{1} f(x) \sin \frac{n\pi x}{1}\, dx \quad \tfrac{1}{2}$$

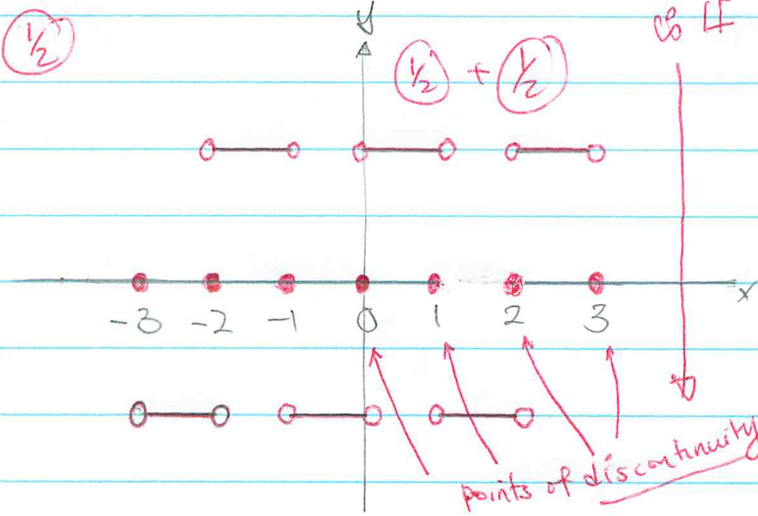
$$= 2\int_0^1 f(x) \sin n\pi x\, dx$$

$$= 2\int_0^1 \sin n\pi x\, dx$$

$$= \frac{-2}{n\pi}\Big[\cos n\pi x\Big]_0^1$$

$$= \frac{+2}{n\pi}\left[1-(-1)^n\right]$$

$$= \begin{cases} \frac{4}{n\pi} & (n \text{ odd}) \\ 0 & (n \text{ even}) \end{cases} \quad \tfrac{1}{2}$$



$$f(x) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left[1-(-1)^n\right] \sin \frac{n\pi x}{1} \quad \tfrac{1}{2}$$

$$= \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi} \sin (2n-1)\pi x$$

10.5.2

$$t u_{xx} + x u_t = 0$$

With $u(x,t) = X(x)\,T(t)$. $\tfrac{1}{2}$

$\tfrac{1}{2}$ $$t X'' T + x X T' = 0$$

$$\frac{X''}{xX} = -\frac{T'}{tT} = -\lambda \quad \text{constant}$$

So
$$X'' + \lambda x X = 0 \quad \tfrac{1}{2}$$
$$T' - \lambda t T = 0 \quad \tfrac{1}{2}$$