HW#9

7.1: 2, 3, 7, 15
7.2: 1, 23, 25

$\boxed{7.1.7}$

$$x_1' = -2x_1 + x_2 \quad (1)$$
$$x_2' = x_1 - 2x_2 \quad (2)$$

$$(1) \Longrightarrow x_2 = x_1' + 2x_1 \quad (3) \qquad \frac{1}{2}$$

$$(2),(3) \Rightarrow \underbrace{(x_1' + 2x_1)'}_{x_2'} = x_1 - 2\underbrace{(x_1' + 2x_1)}_{x_2}$$

$$x_1'' + 2x_1' = -2x_1' - 3x_1 \qquad \frac{1}{2}$$

$$\underbrace{x_1'' + 4x_1' + 3x_1}_{} = 0 \qquad \frac{1}{2}$$

$$r^2 + 4r + 3 = 0 \Rightarrow r = -1, -3 \qquad \frac{1}{2}$$

Thus $x_1 = C_1 e^{-t} + C_2 e^{-3t}$ (4) $\frac{1}{2}$

$$(3),(4) \Longrightarrow x_2 = -C_1 e^{-t} - 3C_2 e^{-3t} + 2C_1 e^{-t} + 2C_2 e^{-3t}$$
$$= C_1 e^{-t} - C_2 e^{-3t} \quad (5) \qquad \frac{1}{2}$$

Given $x_1(0) = 2$, $x_2(0) = 3$:

$$\left.\begin{aligned} C_1 + C_2 &= 2 \\ C_1 - C_2 &= 3 \end{aligned}\right] \longrightarrow \begin{aligned} C_1 &= \frac{5}{2} \quad \frac{1}{2} \\ C_2 &= -\frac{1}{2} \quad \frac{1}{2} \end{aligned}$$

Thus $x_1 = \frac{5}{2}e^{-t} - \frac{1}{2}e^{-3t}$

$\frac{1}{2}$ $= \frac{e^{-t}}{2}\left(5 - e^{-2t}\right)$

$x_2 = \frac{5}{2}e^{-t} + \frac{1}{2}e^{-3t}$

$= \frac{e^{-t}}{2}\left(5 + e^{-2t}\right)$

Note that $5 - e^{-2t} \simeq 5$ for large t.

So $x_1 \simeq \frac{5e^{-t}}{2} \simeq x_2$ for large t.

$x_1'(t) = \frac{e^{-t}}{2}(3e^{-2t} - 5) < 0$

$x_2'(t) = \frac{-e^{-t}}{2}(3e^{-2t} + 5) < 0$

for $t \geq 0$.

x_1 and x_2 are decreasing

Note that $\lim_{t \to \infty} (x_1, x_2) = (0, 0)$.