NICOLE C ALEX **Math 161 - Section 12444 Autumn 2003**

Homework Set Hmwk6 due 11/12/03 at 6:00 AM

This set covers sections 6.1-6.3 and 6.5 of the text.

You may need to give 4 or 5 significant digits for some (floating point) numerical answers in order to have them accepted by the computer.

1.(1 pt) **Note:** You can get full credit for this problem by just answering the last question correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

Consider the indefinite integral $\int x^3 \left(5+5x^4\right)^{12} dx$

Then the most appropriate substitution to simplify this integral is

$u =$ ________

Then $dx = f(x)\,du$ where

$f(x) =$ ________

After making the substitution we obtain the integral $\int g(u)\,du$ where

$g(u) =$ ________

This last integral is: = ________ $+C$

(Leave out constant of integration from your answer.)

After substituting back for u we obtain the following final form of the answer:

= ________ $+C$

(Leave out constant of integration from your answer.)

This is similar to problem 10 on page 264 of the text.

2.(1 pt) Consider the function

$$f(x) = \begin{cases} x & \text{if } x < 1 \\ \frac{1}{x^2} & \text{if } x \geq 1 \end{cases}$$

Evaluate the definite integral.

$$\int_{-1}^{6} f(x)\,dx$$

____

3.(1 pt) Find the area of the region enclosed between $y = 2\sin(x)$ and $y = 3\cos(x)$ from $x = 0$ to $x = 1\pi$. Hint: Notice that this region consists of two parts.

____

4.(1 pt) Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Then find the area of the region.

$x + y^2 = 42, x + y = 0$

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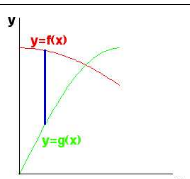
This is similar to problem 25 on page 279 of the text.



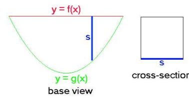
5.(1 pt)

Consider the blue vertical line shown above (click on graph for better view) connecting the graphs $y = g(x) = \sin(4x)$ and $y = f(x) = \cos(3x)$.

Referring to this blue line, match the statements below about rotating this line with the corresponding statements about the result obtained.

____1. The result of rotating the line about the x-axis is
____2. The result of rotating the line about the y-axis is
____3. The result of rotating the line about the line $y = 1$ is
____4. The result of rotating the line about the line $x = -2$ is
____5. The result of rotating the line about the line $x = \pi$ is
____6. The result of rotating the line about the line $y = -2$ is
____7. The result of rotating the line about the line $y = \pi$
____8. The result of rotating the line about the line $y = -\pi$

A. a cylinder of radius $\pi - x$ and height $\cos(3x) - \sin(4x)$
B. a cylinder of radius x and height $\cos(3x) - \sin(4x)$
C. an annulus with inner radius $2 + \sin(4x)$ and outer radius $2 + \cos(3x)$
D. an annulus with inner radius $\pi + \sin(4x)$ and outer radius $\pi + \cos(3x)$
E. an annulus with inner radius $\pi - \cos(3x)$ and outer radius $\pi - \sin(4x)$
F. an annulus with inner radius $1 - \cos(3x)$ and outer radius $1 - \sin(4x)$ is
G. a cylinder of radius $x + 2$ and height $\cos(3x) - \sin(4x)$
H. an annulus with inner radius $\sin(4x)$ and outer radius $\cos(3x)$



6.(1 pt)

The base of a certain solid is the area bounded above by the graph of $y = f(x) = 36$ and below by the graph of $y = g(x) = 16x^2$. Cross-sections perpendicular to the x-axis are squares. (See picture above, click for a better view.)

Use the formula

$$V = \int_a^b A(x)\,dx$$

to find the volume of the formula.

Note: You can get full credit for this problem by just entering the final answer (to the last question) correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

The lower limit of integration is $a =$ ________

The upper limit of integration is $b =$ ________

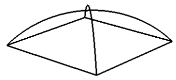
The side s of the square cross-section is the following function of x: ________

$A(x)=$ ________

Thus the volume of the solid is $V =$ ________

See Example 5 on page 283 of the text.

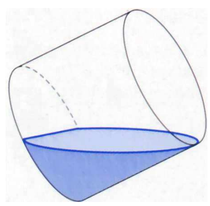
7.(1 pt) The framework of a tent consists of a square base of side $48\sqrt{2}$ and two mutually perpendicular upright ribs formed from circular arcs of radius 73 joining diagonally opposite corners of the base. (Click on picture below for a better view.)



The tent fabric consists of four triangular flaps sewn together so as to fit snugly over the framework. Find the volume enclosed by the tent.

Answer: ______________

8.(1 pt) A cylindrical bucket of radius 12 and height 20 full of water is tipped, and water pours out until the water coincides with a diameter of the base and just touches the rim of the bucket.

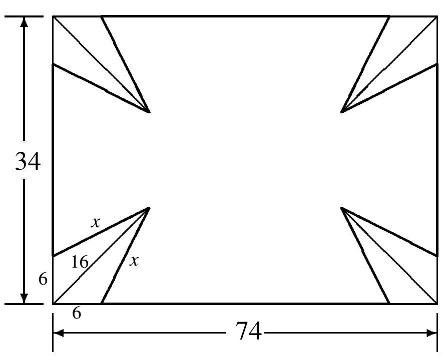


(Click on the picture above for a better view.) What is the volume of water left in the bucket?

____

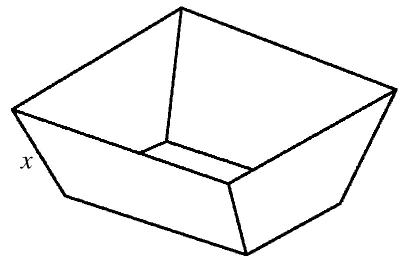
Hint: Compute volume by rectangular cross-sections parallel to the sides of the bucket.

This is similar to problem 36 on page 286 of the text.



9.(1 pt)

Four equal-size kite-shaped wedges are cut from the corners of a 74 by 34 inch rectangle. The short sides of the wedges are 6 inches long, and the (long) diagonals are 16 inches. The sides of the cut rectangle are then folded up and glued along the long sides of the wedges (labelled x in the picture above) to form a trough-shaped box with open top.



What is the volume of the trough-shaped box?

We first compute that

$x =$ ______________

The top of the trough is a rectangle of size ___ (long side) by ___

The bottom of the trough is a rectangle of size ______________ (long side) by ______________

Next noting that the horizontal cross-sections of the trough-shaped box are rectangles, we obtain that the volume is given by

$$V = \int_0^h A(y)\,dy$$

where h is the total (vertical) height of the trough-shaped box, and $A(y)$ denotes the area of the rectangular horizontal cross-section at height y (measured from the bottom of the box).

We find that $h =$ ______________

and that $A(y) =$ ______________

Thus the volume of the trough is $V =$ ______________

10.(1 pt) A soda glass has the shape of the surface generated by revolving the graph of $y = 5x^2$ for $0 \le x \le 1$ about the y-axis. Soda is extracted from the glass through a straw at the rate of $1/2$ cubic inch per second. How fast is the soda level in the glass dropping when the level is 3 inches? (Answer should be implicitly in units of inches per second. Do not put units in your answer. Also your answer should be positive, since we are asking for the rate at which the level DROPS rather than rises.)

answer: ______________

11.(1 pt) To find the length of the curve defined by

$$y = 2x^4 + 13x$$

from the point (0,0) to the point (2,58), you'd have to compute

$$\int_a^b f(x)dx$$

where a=_____,

b=_____,

and f(x) = ______________

12.(1 pt) Find the area of the surface obtained by rotating the graph of

$$y = \sqrt{16x + 80}$$

from $x = -5$ to $x = 11$ about the x-axis.

____

See Example 6 on page 298 of the text.