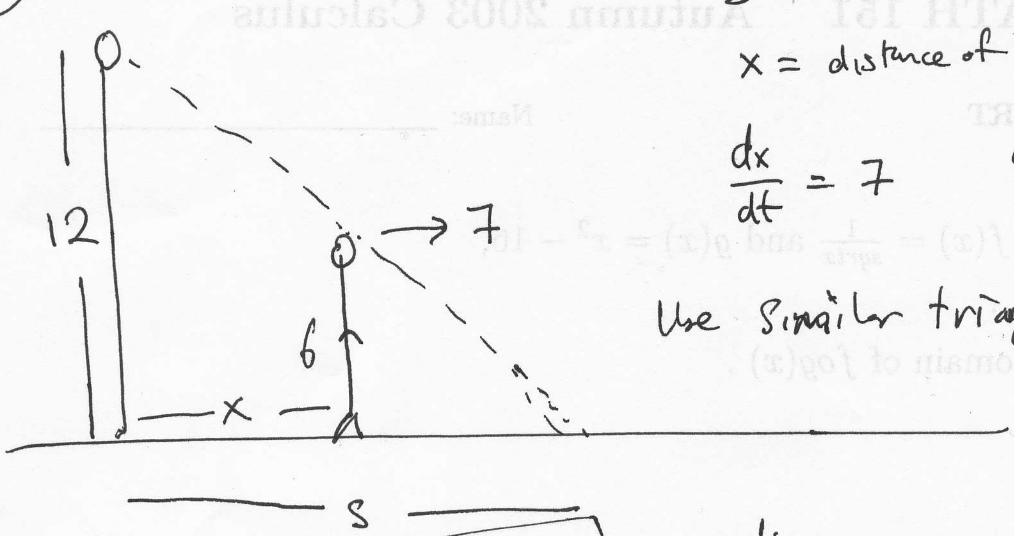
(1)

S = shadow

x = distance of the lady from the pole



$\frac{dx}{dt} = 7 \qquad \left.\frac{ds}{dt}\right|_{x=40} = ?$

Use similar triangles: $\frac{6}{12} = \frac{s-x}{s}$

$S = 2s - 2x$

$s = 2x$

$\boxed{\left.\frac{ds}{dt}\right|_{x=40} = 14} \Longleftarrow \frac{ds}{dt} = 14 \longleftarrow \frac{ds}{dt} = 2\frac{dx}{dt}$

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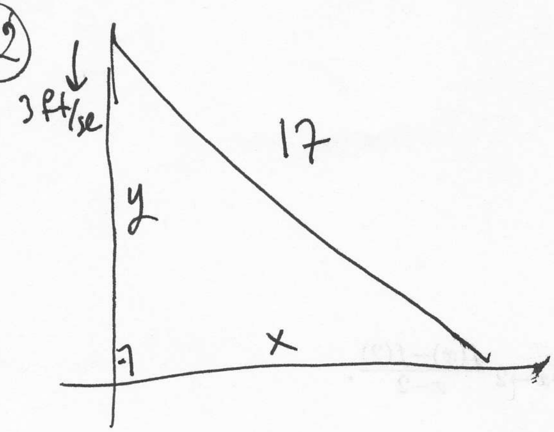
(2)

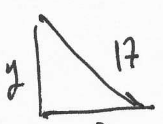


$\frac{dy}{dt} = -3 \qquad \left.\frac{dx}{dt}\right|_{x=8} = ?$

$x^2 + y^2 = 17^2 \Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\Rightarrow \frac{dx}{dt} = -\frac{y}{x}\frac{dy}{dt} \Rightarrow \left.\frac{dx}{dt}\right|_{x=8} = -\frac{15}{8}\cdot(-3)$

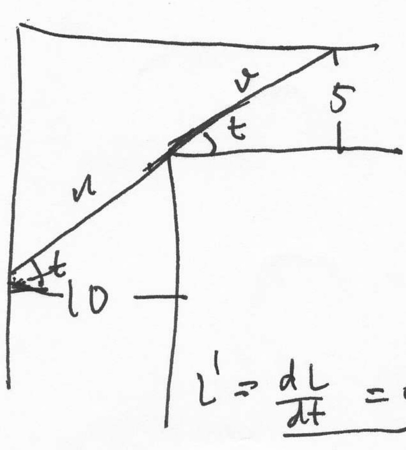
$= \boxed{\frac{45}{8}}$



$\Rightarrow y = \sqrt{17^2 - 8^2}$

$y = 15$

14



$\cos t = \frac{10}{u}, \quad \sin t = \frac{5}{v} \Rightarrow v + u = \frac{10}{\cos t} + \frac{5}{\sin t}$

$L = v + u \Rightarrow \frac{dL}{dt} = \frac{10\sin t}{\cos^2 t} - \frac{5\cos t}{\sin^2 t}$

$= \frac{10\sin^3 t - 5\cos^3 t}{\cos^2 t \cdot \sin^2 t} \qquad 0 < t < \pi/2$

$L' = \frac{dL}{dt} = 0$ if $\quad 10\sin^3 t - 5\cos^3 t = 0$

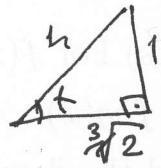
$10\sin^3 t = 5\cos^3 t$

$\tan^3 t = 1/2 \rightarrow \tan t = \frac{1}{\sqrt[3]{2}}$

$\Rightarrow$

Now, this stationary point ($L' = 0$) gives us a rel. <u>min.</u> by (either 1st or 2nd derivative tests)

But then it's a global min since it's unique on the interval $(0, \pi/2)$.



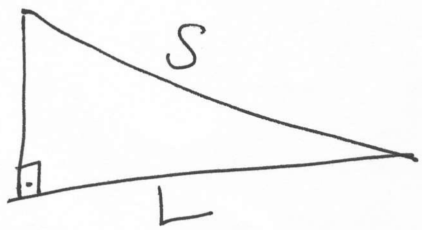
$h = \sqrt{1 + \sqrt[3]{4}}$

$\cos t = \dfrac{\sqrt[3]{2}}{\sqrt{1+\sqrt[3]{4}}}$

$\sin t = \dfrac{1}{\sqrt{1+\sqrt[3]{4}}}$

Then $\boxed{L = \dfrac{10\sqrt{1+\sqrt[3]{4}}}{\sqrt[3]{2}} + \dfrac{5\sqrt{1+\sqrt[3]{4}}}{1}}$

If we are allowed to tilt the pipe vertically then it's enough to assume that we can have the above value without tilting it. i.e



when we tilt it up by x feet.

we want to maximize S. (where $0 \le x \le 11$ corridors are 11 ft high)

$f = S^2 = x^2 + L^2$ $\quad f'(x) = 2x = 0$ if $x = 0$

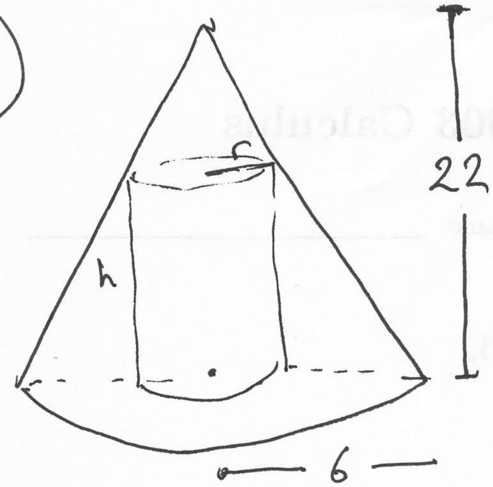
Now $C = \{0, 11\}$ for f. $\quad f(0) = L^2$

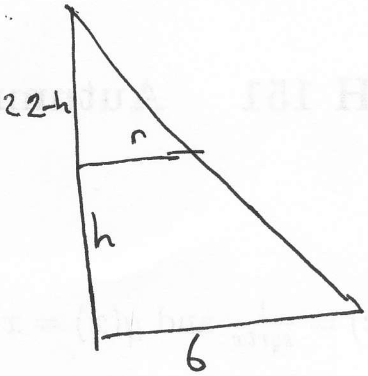
$f(11) = 11^2 + L^2 \Rightarrow$ max

hence $S^* = \sqrt{11^2 + L^2}$ is the maximum.

(Here we mean the above boxed value of L by using it in the answer)

(16)





Similar triangles:

$$\frac{r}{6} = \frac{22-h}{22}$$

$$\boxed{r = \frac{6}{22}(22-h)} \rightarrow \underline{\text{Ans 2}}$$

$$S = \boxed{2\pi r^2 + 2\pi r h} \rightarrow \underline{\text{Ans 1}} \qquad \boxed{0 \le h \le 22} \rightarrow \underline{\text{Ans 4, 5}}$$

$$= \boxed{2\pi\left(\frac{6}{22}\right)^2(22-h)^2 + 2\pi\left(\frac{6}{22}\right)(22-h)\,h} \rightarrow \underline{\text{Ans 3}}$$

$$= \frac{6\pi}{11}\left[\frac{3}{11}(484 + h^2 - 44h) + 22h - h^2\right]$$

$$= \frac{6\pi}{121}\left[3(484 + h^2 - 44h) + 242h - 11h^2\right]$$

$$S'(h) = \frac{6\pi}{121}\left[3(2h - 44) + 242 - 22h\right]$$

$$= \frac{6\pi}{121}\left[6h - 132 + 242 - 22h\right]$$

$$\underline{S' = 0} \quad \text{if} \quad h = \frac{242-132}{22-6} = \frac{110}{16} = \boxed{\frac{55}{8}} \rightarrow \underline{\text{Ans 6}}$$

$$S''(h) = \frac{6\pi}{121}(-18) \Leftrightarrow S''\left(\tfrac{55}{8}\right) < 0 \Rightarrow S \text{ has a rel max at } 55/8$$

$\therefore$ S has a global max at $h = 55/8$

So, $\boxed{S(55/8)}$ gives the final answer