

## Midterm 1 - Review Problems

In each section, the numbers in parentheses represent the problems in the book that are similar to the given ones.

### Section 2.1 – Functions

In problems (1) through (4), find the following values for each function (13-20):

(a)  $f(2)$  (b)  $f(-2)$  (c)  $f(-x)$  (d)  $f(x + 2)$  (e)  $f(x + h)$ .

(1)  $f(x) = x^2 - 4x + 4$

(2)  $f(x) = \frac{x}{x^2+4}$

(3)  $f(x) = |x| - 3$

(4)  $f(x) = \sqrt{x - 5}$

In problems (5) through (8), find the domain, range, and intercepts of the previous functions. (33-48)

In problems (9) through (12), determine whether or not each equation is a function. (21-32)

(9)  $y = x^2 - 3x + 4$

(10)  $x^2 + y^2 = 1$

(11)  $x^2 + y^3 = 4$

(12)  $xy = 1$

### Section 3.1 – Symmetry

In problems (13) through (16), test each equation for symmetry with respect to the  $x$ -axis, the  $y$ -axis, and the origin. Graph each equation using a graphing utility to verify your conclusions. (23-34)

(13)  $y^2 = x^4 + 5x^2 - 17$

(14)  $y = \frac{x}{x^2-9}$

(15)  $y = x^3 + x^2 + x + 1$

(16)  $y^2 = |x|$

### Section 3.2 – Properties of Functions

In problems (17) through (20), use a graphing utility to graph each function over the interval  $(-2, 2)$  and approximate any local minima and maxima. Determine where the function is increasing and decreasing. (51-58)

(17)  $y = x^2$

(18)  $y = (x - 1)^2(x + 1)^2$

(19)  $y = \sqrt{|x|}$

(20)  $y = x(x - 1)(x + 1)$

### Section 3.3 – Library of Functions; Piecewise-Defined Functions

In problems (21) through (24), find the domain of each function, locate any intercepts, graph each function by hand, and then find the range of each function using its graph. (19-30)

$$(21) f(x) = \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases}$$

$$(22) f(x) = \begin{cases} x + 3 & -3 \leq x < 0 \\ 0 & x = 0 \\ -x + 3 & 0 < x \leq 3 \end{cases}$$

$$(23) f(x) = \begin{cases} x & 0 < x < 1 \\ x - 1 & 1 \leq x \leq 2 \end{cases}$$

$$(24) f(x) = \begin{cases} -x - 2 & -3 < x \leq -1 \\ 2 & -1 < x \leq 1 \\ \sqrt{x-1} & x > 1 \end{cases}$$

### Section 3.4 – Transformations

In problems (25) through (28), write the function whose graph is the graph of  $y = x^2$ , but is (17-24):

(25) Shifted left 2 units and up 3 units

(26) Reflected about the  $x$ -axis, then shifted down 3 units

(27) Shifted right 1 unit and vertically stretched by a factor of 3

(28) Shifted 1 unit left then reflected about the  $y$ -axis

In problems (29) through (32), graph each function using transformations. Start with the graph of the basic function and show all stages. Verify your results using a graphing utility. (33-62)

$$(29) f(x) = (x - 1)^3 + 2$$

$$(30) g(x) = \frac{4}{x-1} + 2$$

$$(31) h(x) = x^2 - 4x + 7$$

$$(32) k(x) = -\sqrt{x-2} + 1$$

### Section 4.1 – Power Functions and Models

In problems (33) through (36), use transformations of the graph of  $y = x^4$  or  $y = x^5$  to graph each function. Verify your results using a graphing utility. (1-16)

$$(33) f(x) = (x - 2)^5$$

$$(34) g(x) = 3 - (x + 2)^4$$

$$(35) h(x) = \frac{1}{2}(x + 2)^4 + 3$$

$$(36) k(x) = -(x + 2)^5 - 1$$

## Section 4.2 – Polynomial Functions and Models

In problems (37) through (40), for each polynomial function:

- List each real zero and its multiplicity.
- Determine whether the graph crosses or touches the  $x$ -axis at each  $x$ -intercept.
- Find the power function that the graph of  $f$  resembles for large values of  $|x|$ .
- Use a graphing utility to graph  $f$ .
- Determine the number of turning points on the graph of  $f$ . Approximate the turning points, if any exist, rounded to two decimal places. (19-54)

$$(37) f(x) = 4(x - 2)(x + 1)^2$$

$$(38) f(x) = (x^2 + 1)(x + 1)^2$$

$$(39) f(x) = -x^2(x - 2)(x + 3)^5$$

$$(40) f(x) = 2(x - \sqrt{3})^2(x + 3)^2$$

## Section 4.3 – Rational Functions

In problems (41) through (44), graph each rational function using transformations. (23-34)

$$(41) R(x) = \frac{1}{(x-3)^2}$$

$$(42) R(x) = -\frac{1}{x-1} + 2$$

$$(43) R(x) = \frac{1}{x^2 - 4x + 4}$$

$$(44) R(x) = \frac{x^2 + 4}{x^2}$$

## Section 4.4 – Rational Functions II: Analyzing Graphs

In problems (45) through (48), follow the 8 steps outlined on page 355 to analyze the graph of each function. (1-38)

$$(45) R(x) = \frac{x}{(x-1)(x+1)}$$

$$(46) R(x) = \frac{x^2 + 2}{x - 2}$$

$$(47) R(x) = \frac{x^2 - 4x}{x^2 - 2x + 1}$$

$$(48) R(x) = \frac{x^2 - 4x + 3}{x^2 - 3x + 2}$$

## Section 4.5 – Polynomial and Rational Inequalities

In problems (49) through (52), solve each inequality algebraically. (1-58)

$$(49) (x - 1)(x - 4) < -2$$

$$(50) x^3 \geq 2x^2 - x$$

$$(51) \frac{2x-3}{x+1} > 1$$

$$(52) \frac{2x+5}{x+1} > \frac{x+1}{x-1}$$

### Section – 6.1 – One-to-One Functions; Inverse Functions

In problems (53) through (56), the function  $f$  is one-to-one. Find its inverse and check your answer. State the domain and range of  $f$  and  $f^{-1}$ . By hand, graph  $f$ ,  $f^{-1}$ , and  $y = x$  on the same coordinate axes. (31-42)

$$(53) f(x) = 2x - 1$$

$$(54) f(x) = x^3 + 1$$

$$(55) f(x) = x^2 + 2, x \geq 0$$

$$(56) f(x) = \frac{1}{x-3}$$

### Section 6.2 – Exponential Functions

In problems (57) through (60), use transformations to graph each function. Determine the domain, range, and horizontal asymptote of each function.

(19-36)

$$(57) f(x) = 2^x - 2$$

$$(58) f(x) = -3^{-x}$$

$$(59) f(x) = \frac{1}{3}3^x$$

$$(60) f(x) = 5 + e^{-x}$$

In problems (61) through (64), solve each equation. Verify your solution using a graphing utility. (43-56)

$$(61) 3^{x^3} = 3^x$$

$$(62) 8^{x^2} = 4^x$$

$$(63) e^{x^2} = (e^{4x}) \cdot \frac{1}{e^4}$$

$$(64) \left(\frac{1}{5}\right)^{3-x} = 25$$

### Section 6.3 – Logarithmic Functions

In problems (65) through (68), use transformations to graph each function. Determine the domain, range, and vertical asymptote of each function. (61-84)

$$(65) f(x) = \ln(2 - x)$$

$$(66) f(x) = 4 \log x$$

$$(67) f(x) = \ln(x + 2) - 2$$

$$(68) f(x) = -2 - \log(x - 2)$$

In problems (69) through (72), solve each equation. (85-104)

$$(69) \log_4(2x - 1) = 2$$

$$(70) -\log_3 243 = x^2 + 4x$$

$$(71) e^{-2x} = 4$$

$$(72) \log_6(x^2 + 6x + 41) = 2$$

### Section 6.4 – Properties of Logarithmic Functions

In problems (73) through (76), write each expression as a sum and/or difference of logarithms. Express powers as factors. (25-44)

$$(73) \ln(2xe^x)$$

$$(74) \log\left(\frac{x^2\sqrt{x^2+1}}{(x+3)^4}\right)$$

$$(75) \ln(x\sqrt{x^2-1})$$

$$(76) \log\left(\frac{x^3+2x^2+x}{x^2-3x-4}\right)$$

### Section 6.5 – Logarithmic and Exponential Functions

In problems (77) through (80), solve each equation.

$$(77) \ln x + \ln(x+2) = 4$$

$$(78) 3^{x^2} - 3^{x+1} = -2$$

$$(79) 0.5^{x+1} = 2 \cdot 3^{2x-2}$$

$$(80) \log_{81} x - \log_9 x + \log_3 x = 3$$