Math 366 Exam 1 Name_____

Please show all your work (writing the right answer is not enough for full credit).

(1) (12 pts) Convert each of the following sentences into formal logic statements. Use statement variables such as p, q or r in your answer as well as the logic symbols $\sim$, $\vee$, $\wedge$, $\rightarrow$, and $\leftrightarrow$. Also indicate what the variables represent in each sentence.

(a) Bob studied for the logic exam but he failed.
Solution:
p:Bob studied for the logic exam.
q:Bob passed
"Bob studied for the logic exam but he failed" $\equiv p \wedge \sim q$

(b) Zo sleeps in his cathouse only if I kick him out of bed.
Solution:
p: Zo sleeps in his cat house.
q: I kick Zo out oh bed.
"Zo sleeps in his cathouse only if I kick him out of bed." $\equiv p \rightarrow q$

(c) If I don't finish writing the exam then I may get fired.
Solution:
p: I finish writing the exam.
q: I may get fired.
"If I don't finish writing the exam then I may get fired." $\equiv \sim p \rightarrow q$

(2) (5 points) Use a truth table to verify the following logical equivalence $(p \rightarrow q) \wedge p \equiv (p \wedge q)$.
Solution:

p	q	$p \rightarrow q$	$\boldsymbol{(p \rightarrow q) \wedge p}$	$\boldsymbol{p \wedge q}$
1	1	1	**1**	**1**
1	0	0	**0**	**0**
0	1	1	**0**	**0**
0	0	1	**0**	**0**

Since the last two columns have the same entries, $(p \rightarrow q) \wedge p \equiv (p \wedge q)$.

(3) (10 points) Show that the following argument is valid by converting the argument to an argument form and then using the basic rules of inference:

Either my shirt is black or my cat escaped.
If my cat escaped then I will be sad.
I'm not sad.
$\therefore$ My shirt is black.

Solution:
p: My Shirt is black.
q: My cat escaped.
r: I will be sad.
Hence, our argument has the following formal argument form

$a.(p \vee q) \wedge (\sim p \vee \sim q)$
$b.q \rightarrow r$
$c. \sim r$
$\therefore p$
Then we can its validity is as follows:

1. $p \vee q$ by (a), Specification
2. $\sim q$ by (b),(c), Modus Tollens
3. p by (1),(2), Elimination

Hence, the above argument is valid.

(4) (10 points) Use a truth table to determine whether the following argument form is valid:

$p \vee q$
$p \rightarrow r$
$r \rightarrow \sim p$
$\therefore q$

Solution:

p	q	r	$\sim p$	$\boldsymbol{p \vee q}$	$\boldsymbol{p \rightarrow r}$	$\boldsymbol{r \rightarrow \sim p}$	$\boldsymbol{q}$
T	T	T	F	T	T	F	
T	T	F	F	T	F	T	
T	F	T	F	T	T	F	
T	F	F	F	T	F	T	
F	T	T	T	**T**	**T**	**T**	**T**
F	T	F	T	**T**	**T**	**T**	**T**
F	F	T	T	F	T	T	
F	F	F	T	F	T	T	

Since the truth of the premises implies the truth of the conclusion, this argument is valid.

(5) (10 points) Consider the following Input/Output table for a certain circuit:

P	Q	R	Out
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

(a) Find a Boolean expression for the table.

Solution: $(\sim P \wedge Q \wedge R) \vee (P \wedge \sim Q \wedge R)$

(b) Draw a circuit that obeys the table.
(c) What is the output of the circuit if $P = 0$, $Q = 0$ and $R = 1$? **Solution:** 0

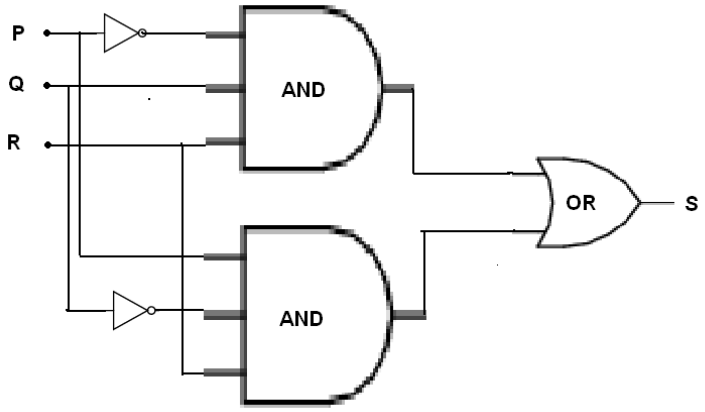

FIGURE 1. Solution for part (b) $(\sim P \wedge Q \wedge R) \vee (P \wedge \sim Q \wedge R)$

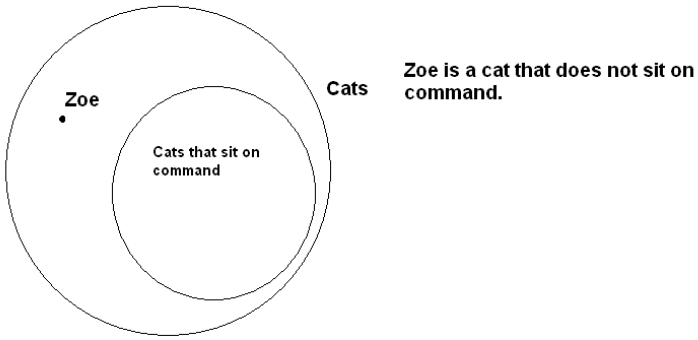

FIGURE 2. Solution Problem.6

(6) (8 points) Show that the following argument is invalid using diagrams:

There are some cats that sit on command.
Zo is a cat.
$\therefore$ Zo sits on command.

(7) (10 points) Translate the following sentences into formal logic statements using predicate symbols and quantifiers. Be sure to indicate the domains of the predicate variables as well as what the predicate symbols represent.

(a) Some dogs don't have any homes.
Solution:

D: Dogs, H: Homes
$P(x, y)$: "x has y as a home"
"Some dogs don't have any homes." $\equiv \exists x \in D \forall y \in H, \sim P(x, y)$

(b) For every two rational numbers p and q, if $p \neq q$ then we can always find a rational number between them.

Solution: $\forall p, q \in \mathbb{Q}, (p \neq q \rightarrow \exists r \in \mathbb{Q}((p < r < q) \vee (q < r < p)))$ or
$P(x, y) : x \neq y$

$Q(x, y, r)$: "r is between x and y", then we have
$\forall p, q \in \mathbb{Q}, (P(p, q) \rightarrow \exists r \in \mathbb{Q}, Q(p, q, r))$

(8) (10 points) Write the negation of the following sentence by first writing the form of the sentence... negating the form... and then writing the statement back in English: "Every donut at the donut shop has some sprinkles on it."

Solution:

X:Donuts at the donut shop
Y: Sprinkles
$A(x, y)$: x has y on it.
Then we have $\forall x \in X, \exists y \in Y, P(x, y)$.
Negation: $\sim (\forall x \in X, \exists y \in Y, P(x, y)) \equiv \exists x \in X, \forall y \in Y, \sim P(x, y)$ which means "Some donuts at the donut shop has no sprinkles on them".

(9) (10 points) Write the argument form for the argument below. If it is valid, state what rule of inference it is. If it is invalid, state what error is being made.
For all integers x, if x is odd then $x + x$ is even.
$6 + 6$ is even.
$\therefore$ 6 is odd.

Solution:

Set
$P(x)$: x is odd.
$Q(x)$: $x + x$ is even.
Then we have
$\forall x \in \mathbb{Z}, (P(x) \rightarrow Q(x))$
$Q(6)$
$\therefore P(6)$
This argument is false by Converse Fallacy Error.

(10) (15 points) Prove or disprove the following statements(you can use behind this page if needed):
a) Any real number has a reciprocal

Counterexample: $x = 0$ does not have a reciprocal since for any $y \in \mathbb{R}$, $xy = 0 \neq 1$. Hence, the statement is false.

b)There are two distinct integers x,y which satisfies $x^2 + y^2 = 169$

Proof: Set $x = 0, y = 13$. Clearly, $0 \neq 13$ and $0^2 + 13^2 = 169$. Hence, the statement is proven.

c)Product of any two odd integers is odd.

Proof. Suppose n, m are [arbitrary but particular] odd integers. By definition of odd, there exist $k, l \in \mathbb{Z}$ such that $n = 2k + 1$ and $m = 2l + 1$. But then we have

$$\begin{aligned} nm &= (2k+1)(2l+1) && \text{by Substitution} \\ &= 4kl + 2k + 2l + 1 && \text{By Basic Algebra} \\ &= 2(2kl + k + l) + 1 && \text{By Basic Algebra} \\ &= 2r + 1 && \text{where } r = 2kl + k + l \end{aligned}$$

Then $r \in \mathbb{Z}$ since sums and products of integers are integers. Hence, nm is odd by definition of odd. $\square$