

### Solutions:

$$(1) \lim_{x \rightarrow 1} \frac{3x^2 - 10x + 8}{5x^2 - 9x + 4} = \lim_{x \rightarrow 1} \frac{(3x-4)(x-2)}{(5x-4)(x-1)} = \frac{(-4)(-1)}{(+1)(0)}$$

$$\text{Hence, } \lim_{x \rightarrow 1^+} \frac{3x^2 - 10x + 8}{5x^2 - 9x + 4} = \frac{(-1)(-1)}{(+1)(0^+)} = +\infty$$

$$(x > 1 \Rightarrow x-1 > 0)$$

$$\lim_{x \rightarrow 1^-} \frac{3x^2 - 10x + 8}{5x^2 - 9x + 4} = \frac{(-1)(-1)}{(+1)(0^-)} = -\infty$$

$$(x < 1 \\ x-1 < 0)$$

$$\lim_{x \rightarrow 1} \frac{3x^2 - 10x + 8}{5x^2 - 9x + 4} = \text{DNE}$$

||

$$(2) \lim_{x \rightarrow 1} \sin \left( \frac{\pi(x^2-1)}{4x-4} \right) = \sin \left( \lim_{x \rightarrow 1} \left( \frac{\pi(x-1)(x+1)}{4(x-1)} \right) \right) = \sin \left( \frac{2\pi}{4} \right) = \frac{\sqrt{2}}{2} //$$

$$(5) \lim_{x \rightarrow \frac{\pi}{2}} (\tan x - \sec x) = \lim_{x \rightarrow \frac{\pi}{2}} \left( \frac{\sin x}{\cos x} - \frac{1}{\cos x} \right) = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x - 1}{\cos x} \cdot \frac{\sin x + 1}{\sin x + 1}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin^2 x - 1}{(\cos x)(\sin x + 1)} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{-\cos^2 x}{(\cos x)(\sin x + 1)} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{-\cos x}{\sin x + 1} = \frac{-0}{2} = 0 //$$

$$(10) \lim_{x \rightarrow 0^+} \frac{|x| - x}{\sqrt[3]{x}} = \lim_{x \rightarrow 0^+} \frac{x - x}{\sqrt[3]{x}} = \lim_{x \rightarrow 0^+} \frac{0}{\sqrt[3]{x}} = \lim_{x \rightarrow 0^+} 0 = 0 //$$

$$\lim_{x \rightarrow 0^-} \frac{|x| - x}{\sqrt[3]{x}} = \lim_{x \rightarrow 0^-} \frac{-x - x}{\sqrt[3]{x}} = \lim_{x \rightarrow 0^-} \frac{-2x}{\sqrt[3]{x}} = \lim_{x \rightarrow 0^-} \frac{-2(\sqrt[3]{x})^3}{\sqrt[3]{x}} = \lim_{x \rightarrow 0^-} \frac{-2(\sqrt[3]{x})^2}{1} = 0 //$$

$$\text{Hence } \lim_{x \rightarrow 0} \frac{|x| - x}{\sqrt[3]{x}} = 0 //$$

$$(13) \lim_{x \rightarrow 0^+} \frac{x}{\sqrt{1-\cos x}} = \lim_{x \rightarrow 0^+} \frac{x \sqrt{1+\cos x}}{\sqrt{1-\cos x} \sqrt{1+\cos x}} = \lim_{x \rightarrow 0^+} \frac{x \sqrt{1+\cos x}}{\sqrt{1-\cos^2 x}} = \lim_{x \rightarrow 0^+} \frac{x \sqrt{1+\cos x}}{|\sin x|}$$

$$= \lim_{x \rightarrow 0} \frac{x \sqrt{1+\cos x}}{\sqrt{\sin^2 x}} = \lim_{x \rightarrow 0} \frac{x \sqrt{1+\cos x}}{|\sin x|} = \begin{cases} \frac{x \sqrt{1+\cos x}}{\sin x} & \text{if } x \rightarrow 0^+ \\ \frac{x \sqrt{1+\cos x}}{-\sin x} & \text{if } x \rightarrow 0^- \end{cases}$$

$$(\sqrt{a^2} = |a|)$$

$$= \begin{cases} \left(\frac{x}{\sin x}\right) (\sqrt{1+\cos x}) & \text{if } x \rightarrow 0^+ \\ \left(\frac{x}{\sin x}\right) (-\sqrt{1+\cos x}) & \text{if } x \rightarrow 0^- \end{cases} = \begin{cases} 1 \cdot \sqrt{2} & \text{if } x \rightarrow 0^+ \\ 1 \cdot (-\sqrt{2}) & \text{if } x \rightarrow 0^- \end{cases}$$

$$\therefore \left( \lim_{x \rightarrow 0^+} \frac{x}{\sqrt{1-\cos x}} = \sqrt{2} \right) \neq \left( \lim_{x \rightarrow 0^-} \frac{x}{\sqrt{1-\cos x}} = -\sqrt{2} \right)$$

Hence limit DNE.

(16)(a) Plug-in

(16)(b) DON'T WORRY ABOUT

DON'T WORRY ABOUT THIS ONE  
AND (17)(i)\*\*