Name:..............................

1. (20 pts) Consider the function

$$f(x) = \begin{cases} \frac{1}{x-2} & \text{if } x > 3 \\ \cos x & \text{if } 0 \le x \le 3 \\ \frac{2}{x+2} & \text{if } x < 0 \end{cases}$$

Find each of the following (if they exist).

(a) $\lim_{x\to 0^-} f(x)$

$$= \lim_{x\to 0^-} \frac{2}{x+2} = \frac{2}{0+2} = 1 \;///$$

(b) $\lim_{x\to 0^+} f(x)$

$$= \lim_{x\to 0^+} \cos x = \cos 0 = 1$$

(c) $f(0)$ $= \cos(0) = 1$

(d) $\lim_{x\to 3} f(x)$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \cos x = \cos 3$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{1}{x-2} = \frac{1}{3-2} = 1$$

$1 \neq \cos 3$ so limit DNE

(e) Find $\lim_{x\to 2} f(x)$

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \cos x = \underline{\underline{\cos 2}} \;///$$

(f) List the discontinuities of $f(x)$

By (a),(b),(c), f is cts at 0

By (d), f is not cts at 3

for $x=-2$ $(-2<0)$, $\frac{2}{x+2}$ is not defined. So, f is not cts at -2

for $x=2$, $\frac{1}{x-2}$ is not defined, but $2<3$ (we need $2>3$). So f is cts at 2.

discontinuities $= \{-2, 3\}$

Name:............................

2. (50 pts) Evaluate the following limits:

(a) $\lim_{x\to 0} \frac{\tan 2x}{x(x-3)} = \lim_{x\to 0} \frac{\left(\frac{\sin 2x}{\cos 2x}\right)}{x(x-3)}$

$= \lim_{x\to 0} \frac{\sin 2x}{2x} \cdot \frac{2}{\cos 2x} \cdot \frac{1}{x-3}$

$= \lim_{x\to 0} \frac{\sin 2x}{2x} \cdot \lim_{x\to 0} \frac{2}{\cos 2x} \cdot \lim_{x\to 0} \frac{1}{x-3} = 1 \cdot 2 \cdot \left(\frac{1}{-3}\right)$

$= \frac{2}{-3}$

(b) $\lim_{x\to -\infty} \frac{3x^3 - 10x + 8}{\sqrt{4x^6 - 9x + 4}}$

$= \lim_{x\to -\infty} \frac{3x^3}{\sqrt{4x^6}} = \lim_{x\to -\infty} \frac{3x^3}{2|x^3|} = \lim_{x\to -\infty} \frac{3\cancel{x^3}}{2(-\cancel{x^3})} = \frac{3}{-2}$ //

Note: $|x^3| = -x^3$ if $x < 0$.

(c) $\lim_{x\to 1} \sin\left(\frac{\pi(x^2-1)}{4x-4}\right)$

$= \sin\left(\lim_{x\to 1} \frac{\pi\cancel{(x-1)}(x+1)}{4\cancel{(x-1)}}\right) = \sin\left(\frac{\pi(1+1)}{4}\right)$

$= \sin\left(\frac{\pi}{2}\right) = 1$ //

(d) $\lim_{x\to 1} \frac{9x^2 - 18x + 8}{x^2 - 3x + 2} = \lim_{x\to 1} \frac{(3x-2)(3x-4)}{(x-2)(x-1)} = \frac{-1}{0}$? Use sided limits...

$\lim_{x\to 1^+} \frac{(3x-2)(3x-4)}{(x-2)(x-1)} = \frac{(1)(-1)}{(-1)(0^+)} = +\infty$

$(x > 1)$
$(x - 1 > 0)$

$\neq$ LIMIT DNE

$\lim_{x\to 1^-} \frac{(3x-2)(3x-4)}{(x-2)(x-1)} = \frac{(1)(-1)}{(-1)(0^-)} = -\infty$

$(x < 1)$