

1. (20 pts)

(a) Find the derivative of the function  $f(x) = \sqrt{5x+4}$  by using the limit definition for derivative.

$$\begin{aligned}
 f'(x) &= \lim_{t \rightarrow x} \frac{\sqrt{5t+4} - \sqrt{5x+4}}{t-x} = \frac{\sqrt{5t+4} + \sqrt{5x+4}}{\sqrt{5t+4} + \sqrt{5x+4}} \\
 &= \lim_{t \rightarrow x} \frac{(5t+4) - (5x+4)}{(t-x)(\sqrt{5t+4} + \sqrt{5x+4})} = \lim_{t \rightarrow x} \frac{5(t-x)}{(t-x)(\sqrt{5t+4} + \sqrt{5x+4})} \\
 &= \frac{5}{2\sqrt{5x+4}}
 \end{aligned}$$

(b) Find the formula for the line tangent to  $f(x)$  at  $x = 1$

$$\begin{aligned}
 m_1 = f'(1) &= \frac{5}{2\sqrt{5+4}} = \frac{5}{6} & f(1) &= \sqrt{5+4} = 3 \\
 y - f(1) &= m_1(x-1) \\
 y - 3 &= \frac{5}{6}(x-1)
 \end{aligned}$$

2. (10 pts) Approximate the value of  $\sqrt[5]{32.01}$  by using Linear Approximation.

$$\begin{aligned}
 f(x) &= \sqrt[5]{x} & f'(x) &= \frac{1}{5} x^{-4/5} & \text{nice point } a &= 32 \\
 f(32) &= 2 & f'(32) &= \frac{1}{5} (32)^{-4/5} = \frac{1}{80}
 \end{aligned}$$

$$\begin{aligned}
 \sqrt[5]{32.01} &\approx f(a) + (32.01 - a) \cdot f'(a) \\
 &= 2 + (0.01) \frac{1}{80} = 2.000125
 \end{aligned}$$

Name:.....

3. (40pts) Find the following derivatives [State the derivative rules that you used]

(a)  $f(x) = \cot(x^3 + e^x) + 5x$

$$f'(x) = \underbrace{-\csc^2(x^3 + e^x) \cdot (3x^2 + e^x)}_{CR} + 5$$

$$\underbrace{\hspace{10em}}_{SR}$$

(b)  $y = (\sin(\cos x))(\ln(x^2))$

$$\frac{dy}{dx} = \underbrace{\cos(\cos x) \cdot (-\sin x)}_{CR} \ln(x^2) + \underbrace{\left(\frac{1}{x^2} \cdot 2x\right)}_{CR} \sin(\cos x)$$

$$\underbrace{\hspace{15em}}_{PR}$$

(c)  $g(x) = \frac{5x^2 + 10x}{\sin^2 x}$

$$g'(x) = \frac{\overbrace{(10x + 10)}^{CR}}{\sin^2 x} - \frac{(2 \sin x \cos x)(5x^2 + 10x)}{\sin^4 x} \quad ] \text{QR}$$