Name:..............................

1. Let $f(x) = \dfrac{1}{x^2+1}$, $f'(x) = \dfrac{-2x}{(x^2+1)^2}$, $f''(x) = \dfrac{2(3x^2-1)}{(x^2+1)^3}$

(a) (20 pts) Find the set on which $f(x)$ is increasing and the set on which $f(x)$ is decreasing.[Don't forget to check the domain for your own good]

DOMAIN $= (-\infty, \infty)$

$f' = 0$: $x = 0$ $\quad$ f' DNE: never

$(-1) \quad 0 \quad (1)$

$f'(-1) > 0$; $f'(1) < 0$

f' $+ + + + 0 - - - - -$

f $\nearrow \quad \searrow$

f is $\uparrow$ on $(-\infty, 0]$

$\downarrow$ on $[0, \infty)$

(b) (10 pts) Find the critical points and local(or relative) extrema of $f(x)$ (if any).

$C = \{0\}$ $\quad$ f' changes $(+) \to (-)$ around 0

By 1st Der Test, f has a local max at $(0, 1)$

(c) (20 pts) Find the set on which $f(x)$ is concave up and the set on which $f(x)$ is concave down. Find the inflection points [if any].

$f'' = 0$: $3x^2 - 1 = 0$

$x = \mp \dfrac{1}{\sqrt{3}}$

f'' DNE: never

$(-1) \quad -1/\sqrt{3} \quad (0) \quad 1/\sqrt{3} \quad (1)$

$f''(-1) > 0$; $f''(0) < 0$; $f''(1) > 0$

f'' $+ + + + 0 - - - 0 + + + + +$

f $\cup \quad \cap \quad \cup$

f'' changes sign around $-\dfrac{1}{\sqrt{3}}$ and $\dfrac{1}{\sqrt{3}}$ and they are both in the domain

f is $\cup$ on $\left(-\infty, -\dfrac{1}{\sqrt{3}}\right)$ and $\left(\dfrac{1}{\sqrt{3}}, \infty\right)$

f is $\cap$ on $\left(-\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}\right)$

so $I = \left\{-\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}\right\}$

2. (25 pts) Find THE maximum and THE minimum of $f(x) = (x^2-5x+4)^{1/3}$ on the interval [0,3].[Do not forget to check the continuity of your function on the given interval.]

$u = x^2 - 5x + 4$ is cts everywhere
$u^{1/3}$ is cts everywhere
$\therefore$ So, $(x^2-5x+4)^{1/3}$ is cts everywhere

Hence by Min-Max Thm, f has the max and min on $[0,3]$

$$f'(x) = \frac{1}{3}(x^2-5x+4)^{-2/3}(2x-5) = \frac{2x-5}{3\left[(x-4)(x-1)\right]^{2/3}}$$

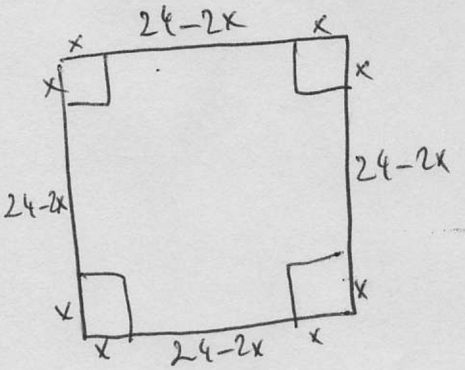
$\underline{f'=0}$: $x = \frac{5}{2}$ $\qquad$ $\underline{f' \text{ DNE}}$: $x = 1$, 4 not in $[0,3]$ $\qquad$ $C = \{0, 3, 1, \frac{5}{2}\}$

$f(0) = \sqrt[3]{4}$ $\qquad$ $f(1) = 0$ $\qquad$ $f(3) = -\sqrt[3]{2}$ $\qquad$ $f(\frac{5}{2}) = -\sqrt[3]{\frac{9}{4}}$

f has its MAX at $(0, \sqrt[3]{4})$

MIN at $(\frac{5}{2}, -\sqrt[3]{\frac{9}{4}})$ by

3. (25 pts) A square piece of tin 24 inch on each side is to be made into an open-top box by cutting a small square from the corners and bending up the flaps to form the sides. How large a square should be cut from each corner to make the volume of the box as large as possible?



$0 \le x \le 12$

$$V = x(24-2x)^2 = 4x(x^2 - 24x + 144)$$
$$= 4(x^3 - 24x^2 + 144x)$$
$$V' = 4(3x^2 - 48x + 144)$$
$$= 12(x^2 - 16x + 48)$$
$$= 12(x-4)(x-12)$$

$\underline{V' = 0}$: $x = 4, 12$

$\underline{V' \text{ DNE}}$: never

$C = \{0, 4, 12\}$

$V(0) = 0$

$V(4) = 4(16)^2 = 1024$

$V(12) = 0$

So, V is maximum if $x = 4$

hence squares must be 4×4.