

MATH 153 Summer 2008 Calculus

Lecturer: Oguz KURT
Quiz 2

Name: _____

- (10 points) (a) Find the slope of the tangent to the astroid $x = \cos^3 t$, $y = \sin^3 t$ in terms of t .
(b) At what points is the tangent horizontal or vertical?
(c) At what points does the tangent have slope 1 or -1 ?

Solution:

$$\text{a) } \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3 \sin^2 t \cos t}{-3 \cos^2 t \sin t} = -\frac{\sin t}{\cos t}$$

b) *horizontal tangent line when $dy/dt = 0$ but $dx/dt \neq 0$: $\sin t = 0$ when $t = 0, \pi$*

vertical tangent line when $dx/dt = 0$ but $dy/dt \neq 0$: $\cos t = 0$ when $t = \frac{\pi}{2}, \frac{3\pi}{2}$
(Note that these are points where dy/dt and dx/dt are both zero. Yet, in the actual form of dy/dx , many cancellations happen. Since the slope is a limit, those cancellations can be ignored.)

$$\text{c) } \frac{dy}{dx} = -\tan t = 1 \text{ when } t = \frac{3\pi}{4}, \frac{7\pi}{4}$$
$$\frac{dy}{dx} = -\tan t = -1 \text{ when } t = \frac{\pi}{4}, \frac{5\pi}{4}$$

- (10 points) Sketch the curve with the polar equation $r = 1 + 2 \cos(\frac{\theta}{2})$.

