MATH 153 Summer 2005 Calculus

Lecturer: Oguz KURT Name: _____

Quiz 1

1. Check the convergence or divergence of the following series and find their value, if possible. Show your work!

[a] (7 points) $\sum_{k=1}^{\infty} \frac{2^k}{k!}$ = convergent by ratio test

$$\lim_{k\to\infty}\left(\frac{2^{k+1}}{(k+1)!}\cdot\frac{k!}{2^k}\right)=\lim_{k\to\infty}\frac{2}{k+1}=0<1$$

[b] (7 points) $\sum_{k=2}^{\infty}\left(\frac{5\times 7^k}{8^{k-1}}+\frac{3\times 4^{k-1}}{5^k}\right) = 5(7)\sum_{k=2}^{\infty}\frac{7^{k-1}}{8^{k-1}} + \frac{3}{5}\sum_{k=2}^{\infty}\frac{4^{k-1}}{5^{k-1}}$

$$= 35\left(\frac{7}{8}\right)\sum_{n=0}^{\infty}\left(\frac{7}{8}\right)^n + \left(\frac{3}{5}\right)\left(\frac{4}{5}\right)\sum_{n=0}^{\infty}\left(\frac{4}{5}\right)^n \qquad \frac{7}{8}<1,\ \frac{4}{5}<1$$

$$= (35)\left(\frac{7}{8}\right)\left(\frac{1}{1-\frac{7}{8}}\right) + \left(\frac{12}{25}\right)\left(\frac{1}{1-\frac{4}{5}}\right)$$

Note $\sum_{k=2}^{\infty}\left(\frac{7}{8}\right)^{k-2} = \sum_{n=0}^{\infty}\left(\frac{7}{8}\right)^n$ if we set $k-2=n$

[c] (6 points) $\sum_{k=1}^{\infty}\underbrace{\frac{n-1}{n^2+n+6}}_{a_n}$ set $b_n = \frac{n}{n^2} = \frac{1}{n}$

$$\lim_{n\to\infty}\frac{a_n}{b_n}=\lim_{n\to\infty}\frac{n-1}{n^2+n+6}\cdot\frac{n^2}{n}=\lim_{n\to\infty}\frac{n^3-n^2}{n^3+n^2+6n}=1$$

so $\sum a_n$, $\sum b_n$ converge or diverge together

b_n: $\sum b_n = \sum\frac{1}{n}$ diverges $\Rightarrow$ $\sum\frac{n-1}{n^2+n+6}$ diverges