

# Midterm 2

1. (25 pts) Find the equation of the ellipse with foci at  $(-2, 2)$  and  $(2, 2)$  that passes through the origin and sketch its graph.

foci at  $(-2, 2), (2, 2) \Rightarrow$  center at  $\left(\frac{-2+2}{2}, \frac{2+2}{2}\right)$   
 $\parallel$   
 $(0, 2)$

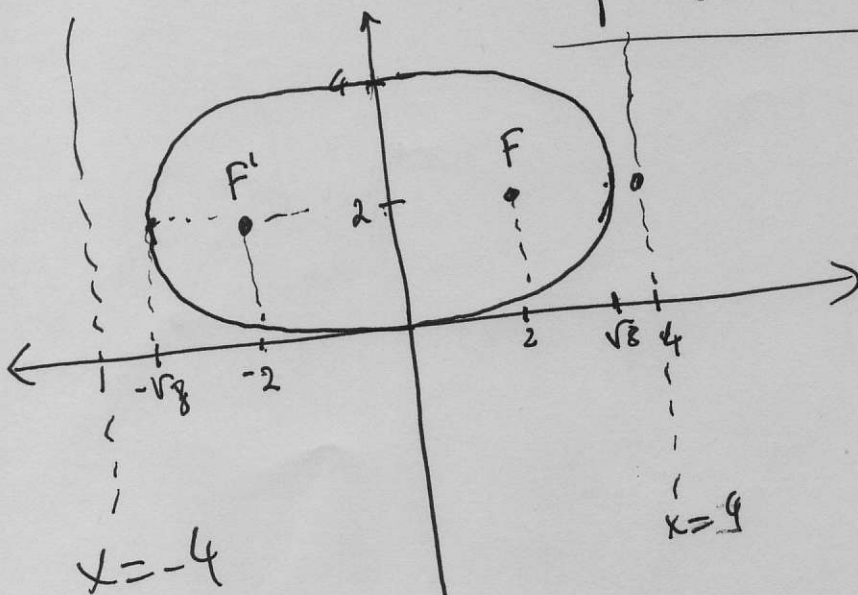
~~vertices~~  
 touches origin  $\Rightarrow b = 2$  for  $\frac{(y-2)^2}{b^2} + \frac{x^2}{a^2} = 1$

$c = \frac{\text{distance between two foci}}{2} = \frac{4}{2} = 2$

$a^2 = b^2 + c^2 \Rightarrow a^2 = 4 + 4 = 8$   
 $a = \sqrt{8}$

$\Rightarrow$  eqn:  $\frac{(y-2)^2}{4} + \frac{x^2}{8} = 1$

~~$c = ae$~~   
 $2 = \sqrt{8}e$   
 $\Rightarrow e = \frac{\sqrt{2}}{2}$



directrix:  $x = \bar{r} \frac{a}{e}$   
 $= \bar{r} \frac{\sqrt{8}}{(\sqrt{2}/2)}$   
 $= \bar{r} 4$

2. (30 pts) Sketch the graph of the equation  $16x^2 - 9y^2 + 192x + 90y + 207 = 0$ . If it is a conic, then name the conic and give all the information about its focus, directrix, eccentricity and asymptotes if possible.

$$16x^2 - 9y^2 + 192x + 90y + 207 = 0$$

$$16(x^2 + 12x + 36 - 36) - 9(y^2 - 10y + 25 - 25) + 207 = 0$$

$$16(x+6)^2 - 9(y-5)^2 - 16(36) + 9(25) + 207 = 0$$

$$16(x+6)^2 - 9(y-5)^2 = 144$$

$$\boxed{\frac{(x+6)^2}{9} - \frac{(y-5)^2}{16} = 1}$$

horizontal hyperbola centered at  $(-6, 5)$ . with  $a=3$   
 $b=4$

$$c^2 = a^2 + b^2 = 25$$

$$\boxed{e=5}$$

$$ae = c$$

$$3e = 5$$

$$\boxed{e = \frac{5}{3}}$$

$$\left[ \begin{array}{l} \text{foci at } (-6, 5) \text{ and } (5, 5) \\ (-6, 5) + (5, 0) = (-1, 5) \\ (-6, 5) + (-5, 0) = (-11, 5) \end{array} \right.$$

~~directrix~~  
[asymptotes]  
 $y-5 = \pm \frac{4}{3}(x+6)$

[directrix]

$$x+6 = \mp \frac{3}{(5/3)}$$

$$x = -\frac{21}{5}, x = -\frac{39}{5}$$

