$\boxed{\text{HW 4.4}}$

~~4.4.~~

4.4.32: Let x be the length of edges of the cube.

$$\begin{pmatrix}\text{surface}\\ \text{area}\\ \text{of cube}\end{pmatrix} = 6x^2 \Rightarrow 0 \le x \le \frac{1}{\sqrt{6}}$$

$$\begin{pmatrix}\text{The surface area}\\ \text{of sphere}\end{pmatrix} = 4\pi r^2 \quad \text{so} \quad S = 6x^2 + 4\pi r^2 = 1$$

$$r = \sqrt{\frac{1-6x^2}{4\pi}}$$

$$V = x^3 + \frac{4}{3}\pi r^3 = x^3 + \frac{4}{3}\pi\left(\frac{1-6x^2}{4\pi}\right)^{3/2}$$

$$V' = 3x^2 + \cancel{\tfrac{4}{3}\pi}\left(\frac{3}{2}\left(\frac{1-6x^2}{4\pi}\right)^{1/2}\left(-\frac{12x}{\cancel{4\pi}}\right)\right)$$

$$= 3x^2 - 6x\sqrt{\frac{1-6x^2}{4\pi}} = 3x\left(x - 2\sqrt{\frac{1-6x^2}{4\pi}}\right)$$

$\underline{V' = 0}$ if $\boxed{x=0}$ or $x - 2\sqrt{\frac{1-6x^2}{4\pi}} = 0$

$$[x]^2 = \left[2\sqrt{\frac{1-6x^2}{4\pi}}\right]^2$$

$$x^2 = 4\left(\frac{1-6x^2}{4\pi}\right) \to \pi x^2 = 1 - 6x^2$$

$$x^2 = \frac{1}{\pi+6}$$

$$x = \mp\sqrt{\frac{1}{\pi+6}}$$

Only $x = 0, \sqrt{\frac{1}{\pi+6}}$ falls in $\left[0, \frac{1}{\sqrt{6}}\right]$

$$V(0) = \frac{1}{6\sqrt{\pi}} \approx 0.094\,m^3$$

$$V\left(\sqrt{\frac{1}{\pi+6}}\right) = (6+\pi)^{-3/2} + \frac{1}{6\sqrt{\pi}}\left(1 - \frac{6}{6+\pi}\right)^{3/2} \approx 0.055\,m^3$$

$$V\left(\frac{1}{\sqrt{6}}\right) = \frac{1}{6\sqrt{6}} \quad \text{is in between them}$$

max volume happens when $x = 0$ $\left(r = \frac{1}{2\sqrt{\pi}}\right)$

min volume happens when $x = \frac{1}{\sqrt{\pi+6}}$ $\left(r = \frac{1}{2\sqrt{6+\pi}} \approx 0.165\,m\right)$

HW 4.4

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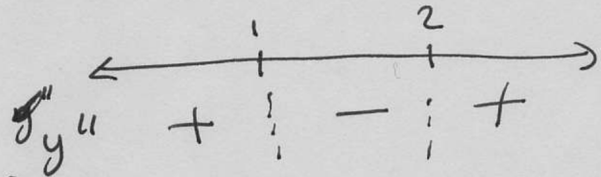
$y' = dy/dx = 4x^3 - 18x^2 + 24x - 3$

$y'' = d^2y/dx^2 = 12x^2 - 36x + 24 = 12(x^2 - 3x + 2)$
$= 12(x-2)(x-1)$

$y'' = \dfrac{d^2y}{dx^2} = 0$ when $x = 1, 2$



so I.P. at $(1, f(1)) = (1, 5)$
$(2, f(2)) = (2, 11)$

Slope at $x=1$: $\left.\dfrac{dy}{dx}\right|_{x=1} = 7$

Tangent line at $x=1$: $\boxed{y - 5 = 7(x-1)}$