Solutions: In-Class Work 1

1.

$$(x+h)^2 = x^2 + 2xh + h^2$$
$$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$$
$$(x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$
$$(x+h)^5 = x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5$$

2.

$$\frac{(x+h)^2 - x^2}{h} = 2x + h$$
$$\frac{(x+h)^3 - x^3}{h} = 3x^2 + 3xh + h^2 = 3x^2 + h(3x+h)$$
$$\frac{(x+h)^4 - x^4}{h} = 4x^3 + 6x^2h + 4xh^2 + h^3 = 4x^3 + h(6x^2 + 4xh + h^2)$$
$$\frac{(x+h)^5 - x^5}{h} = 5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4 = 5x^4 + h(10x^3 + 10x^2h + 5xh^2 + h^3)$$

3.

$$\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} 2x + h = 2x$$
$$\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h\to 0} 3x^2 + h(3x+h) = 3x^2$$
$$\lim_{h\to 0} \frac{(x+h)^4 - x^4}{h} = \lim_{h\to 0} 4x^3 + h(6x^2 + 4xh + h^2) = 4x^3$$
$$\lim_{h\to 0} \frac{(x+h)^5 - x^5}{h} = \lim_{h\to 0} 5x^4 + h(10x^3 + 10x^2h + 5xh^2 + h^3) = 5x^4$$

4.

$$(x+h)^n = x^n + nx^{n-1}h + h^2(Ax^{n-2} + ...)$$
$$\frac{(x+h)^n - x^n}{h} = nx^{n-1} + h(Ax^{n-2} + ...)$$
$$\lim_{h\to 0} \frac{(x+h)^n - x^n}{h} = \lim_{h\to 0} nx^{n-1} + h(Ax^{n-2} + ...) = nx^{n-1}$$

5.

$$\sqrt[3]{x+h} - \sqrt[3]{x} = (\sqrt[3]{x+h} - \sqrt[3]{x})\frac{\sqrt[3]{(x+h)^2} + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x^2}}{\sqrt[3]{(x+h)^2} + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x^2}} = \frac{h}{\sqrt[3]{(x+h)^2} + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x^2}}$$

because we know $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ and if we set $a^3 = x + h$ ($a = \sqrt[3]{x+h}$) and $b^3 = x$ ($a = \sqrt[3]{x}$) we will get $h = (x+h) - x = (\sqrt[3]{x+h} - \sqrt[3]{x})(\sqrt[3]{(x+h)^2} + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x^2})$

6.

$$\lim_{h\to 0} \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h} = \lim_{h\to 0} \frac{1}{\sqrt[3]{(x+h)^2} + \sqrt[3]{x+h}\sqrt[3]{x} + \sqrt[3]{x^2}} = \frac{1}{\sqrt[3]{x^2} + \sqrt[3]{x}\sqrt[3]{x} + \sqrt[3]{x^2}} = \frac{1}{3\sqrt[3]{x^2}} = \frac{1}{3}x^{-\frac{2}{3}}$$