

Math 151

Winter Quarter 2006

INCLASS WORK # 2

Attach all Work and Sign Off Below!

#	Last Name	First Name	Last 4 digits of SSN
1)			
2)			
3)			
4)			

[This is essentially Problem 44 from Section 3.5]

A rotating wheel of radius 1 is moving a piston up and down via a rod of length 5 that is attached point P to the wheel and at point Q to the piston. See picture.

The goal is to compute the velocity of the piston from the speed of rotation.

1. Introduce a (dependent) variable for the height of the piston. (distance Q to O).

Of what (independent) variable should this be a function if you want to compute speed as a derivative?

Of what (independent) variable should this be a function if you want to compute it from the depicted geometry?

2. Find the height as a function of the angle θ .
3. Suppose the wheel is turning at 30 rotations per second.

What is θ as a function of time t ? (assume $\theta = 0$ at $t = 0$).

4. In this case (3.) find also the height as a function of t .
5. Still for (3.) compute the (vertical) velocity of the piston for an arbitrary time t .
6. The rotation is not uniform, that is, the rotation of the wheel may accelerate or decelerate. Hence θ is a general function $\theta(t)$ on time t .
7. Write and explain the height as a function in t as a composition of functions, where the inner composite is $\theta(t)$.
8. Find a general expression for the time derivative of the height function that involves $\theta(t)$ and $\theta'(t)$.
9. Suppose we have at time $t_o = \sqrt{17}$ that $\theta(\sqrt{17}) = \frac{\pi}{3}$ and $\theta'(\sqrt{17}) = 23$. What is the velocity of piston at $t_o = \sqrt{17}$?
10. Is it really necessary to know that the value of the time $t_o = \sqrt{17}$, or is it enough to give the values $\theta(t_o)$ and $\theta'(t_o)$ at a time instance t_o with unspecified value?
11. The above is an example of a "Related Rates" problem (here about the relation between the rate by which the piston height changes and the rate with which the wheel rotates). In calculations the time parameter is often suppressed as an argument (write θ instead of $\theta(t)$) and Leibniz notation is used (write $\frac{d\theta}{dt}$ instead of $\theta'(t)$).

Rewrite the calculations you did above in this notation.

