Feb 13th, 2006

Math 151

Winter 2006

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Name: ....

SSN (last 4 digit): XXX-XX-................

Check Your Recitation Class:

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☐ 9:30	☐ 9:30	☐ 9:30	☐ 11:30	☐ 11:30	☐ 11:30
☐ 10:30	☐ 10:30	☐ 10:30	☐ 12:30	☐ 12:30	☐ 12:30

TEST 2 *Form E*

• You have 48 minutes to answer the following 4 questions. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

• Write your name and the last four digits of your social security number on the top of this page. and check off your recitation section by time and TA.

• Check that no pages of your exam are missing. This exam has 7 printed and numbered pages.

• No books or notes are allowed on this exam! Calculators are permitted only as stated in the "Calculator Policy" of the course. Electronic communication devices of any kind are prohibited.

• Show all work! Even correct answers without work may result in point deductions. Do not simplify arithmetical expression such as $\frac{\sqrt{7}}{3} - \frac{5}{\sqrt{11}}$.

• Plan your time wisely! It is a good idea to skim over all of the questions before you start. Do not spend too much time on one particular problem.

Good Luck ! 1

Question	Max. Points	Score
1	15	
2	40	
3	20	
4	25	
Total	100	

1) [15 Points] Consider the following function:

$$f(x) \;=\; x^3 \,-\, 3x^2 \,-\, 7x \,+ 9$$

Find **all** points x where the tangent to the graph of $f(x)$ has slope $m = 17$.

Solution:

Require $f'(x) \;=\; 3x^2 \,-\, 6x \,-\, 7 \stackrel{?}{=} 17$.
True iff $3x^2 \,-\, 6x \,-\, 24 \;=\; 0$. That is, iff, $3(x+2)(x-4) \;= 0$.
Hence, tangent slopes occur at $x = -2$ and $x = 4$.

2) [40 Points]
Find the following derivatives:

Show all work! Do not simplify algebraic or numerical expressions.

(a) For the function $g(x) = \dfrac{9}{x^3} + 4\cos(x)$ compute the derivative $g'(x)$.

Solution:

$$g'(x) = 9\tfrac{d}{dx}(x^{-3}) + 4\tfrac{d}{dx}\cos(x) = 9(-3)x^{-3-1} + 4(-\sin(x)) = -\frac{27}{x^4} - 4\sin(x)$$

(b) For $h(t) = \sqrt[3]{t+3} \cdot \tan(t) = (t+3)^{\frac{1}{3}} \cdot \tan(t)$ determine the derivative $D_t h(t)$.

Solution:

$$\begin{aligned} h'(t) &= \tfrac{d}{dt}((t+3)^{\frac{1}{3}}) \cdot \tan(t) + (t+3)^{\frac{1}{3}} \cdot \tfrac{d}{dt}\tan(t) \\ &= \tfrac{1}{3}((t+3)^{\frac{1}{3}-1}) \cdot \tan(t) + (t+3)^{\frac{1}{3}} \cdot (\sec^2(t)) \\ &= \tfrac{1}{3}(t+3)^{-\frac{2}{3}} \cdot \tan(t) + (t+3)^{\frac{1}{3}} \cdot \sec^2(t) \end{aligned}$$

Show all work! Do not simplify algebraic or numerical expressions.

(c) Find the derivative $\dfrac{d}{dp}\left(\left(\dfrac{9+\cot(p^4)}{7+p^2}\right)^{463}\right)$

Solution:

$$\frac{d}{dp}\left(\left(\frac{9+\cot(p^4)}{7+p^2}\right)^{463}\right) = 463\left(\frac{9+\cot(p^4)}{7+p^2}\right)^{463-1}\frac{d}{dp}\left(\frac{9+\cot(p^4)}{7+p^2}\right) =$$

$$= 463\left(\frac{9+\cot(p^4)}{7+p^2}\right)^{462}\frac{\frac{d}{dp}(9+\cot(p^4))(7+p^2) - (9+\cot(p^4))\frac{d}{dp}(7+p^2)}{(7+p^2)^2}$$

$$= 463\left(\frac{9+\cot(p^4)}{7+p^2}\right)^{462}\frac{(-\csc^2(p^4))\frac{d}{dp}p^4(7+p^2) - (9+\cot(p^4))2p^1}{(7+p^2)^2}$$

$$= 463\left(\frac{9+\cot(p^4)}{7+p^2}\right)^{462}\frac{-4\csc^2(p^4)p^3(7+p^2) - 2(9+\cot(p^4))p^1}{(7+p^2)^2}$$

(d) Compute the second derivative $\dfrac{d^2}{d\theta^2}\left(\sin(\theta^5)\right)$.

Solution: $\dfrac{d}{d\theta}\sin(\theta^5) = \cos(\theta^5)\dfrac{d}{d\theta}\theta^5 = 5\cos(\theta^5)\theta^4$

$$\frac{d^2}{d\theta^2}\sin(\theta^5) = \frac{d}{d\theta}\left(5\cos(\theta^5)\theta^4\right) = 5\frac{d}{d\theta}(\cos(\theta^5))\theta^4 + 5\cos(\theta^5)\frac{d}{d\theta}(\theta^4)$$

$$= -5\sin(\theta^5)\tfrac{d}{d\theta}(\theta^5)\theta^4 + 5\cos(\theta^5)(4\theta^3) = -5\sin(\theta^5)(5\theta^4)\theta^4 + 5\cos(\theta^5)(4\theta^3)$$

$$= -25\sin(\theta^5)\theta^8 + 20\cos(\theta^5)\theta^3$$

3)[20 Points] Suppose $f(x)$ and $g(x)$ are differentiable functions on $[0,10]$.

x	1	2	5	8
$f(x)$	2	1	5	2
$f'(x)$	$\frac{8}{\pi}$	-11	17	$-\frac{1}{\sqrt{5}}$
$g(x)$	2	1	1	8
$g'(x)$	12	$\frac{1}{\sqrt{7}}$	$\frac{\pi}{5}$	$-\frac{1}{\pi}$

Assume further that the values of these two functions as well as the values of their derivatives at the points $x =$ 1, 2, 5, and 8 are given by the values in the table on the right. (For example, $f'(2) = -11$)

Compute the values of the following derivatives of functions obtained from $f(x)$ and $g(x)$:

Show all steps and all work!

Do not simplify purely numerical expressions!

(a) For $p(x) = f(x)g(x)$ determine $p'(8)$:

Solution:

$$p'(x) = f'(x)g(x) + f(x)g'(x)$$

$$p'(8) = f'(8)g(8) + f(8)g'(8)$$

$$= (-\tfrac{1}{\sqrt{5}}) \cdot (8) + (2) \cdot (-\tfrac{1}{\pi}) .$$

(b) For $h(x) = (f \circ g)(x) = f(g(x))$ determine $h'(5)$:

Solution:

$$h'(x) = f'(g(x))g'(x)$$

$$h'(5) = f'(g(5))g'(5)$$

$$= f'(1) \cdot (\tfrac{\pi}{5})$$

$$= (\tfrac{8}{\pi}) \cdot (\tfrac{\pi}{5})$$

(c) For $q(x) = (g \circ f)(x) = g(f(x))$ determine $q'(2)$:

Solution: $q'(x) = g'(f(x))f'(x)$

$$q'(2) = g'(f(2))f'(2)$$

$$= g'(1) \cdot (-11)$$

$$= (12) \cdot (-11)$$

4)[25 Points] **Show all Work!**

Consider the curve in the xy-plane described by the equation

$$y^3 + 5yx^7 - 42 = 0$$

(a) Verify explictly that the point $P = (1, 3)$ lies on this curve.

Solution:

Plug coordinates $x = 1$ and $y = 3$ into equation:

$(3)^3 + 5 \cdot (3) \cdot 1^7 - 42 = 27 + 15 - 42 = 0$

(b) Find an expression in x and y for $\frac{dy}{dx}$ using implict differentiation.

Solution:

Differentiate Equation:

$$0 = \frac{d}{dx}\left(y^3 + 5yx^7 - 42\right) = \frac{d}{dx}(y^3) + 5\frac{d}{dx}(yx^7) =$$

$$= 3y^2\frac{dy}{dx} + 5\frac{dy}{dx}x^7 + 5y(7x^6) = (3y^2 + 5x^7)\frac{dy}{dx} + 35yx^6 = 0$$

Thus $$\frac{dy}{dx} = -\frac{35yx^6}{3y^2 + 5x^7}$$

(c) Find the equation of the tangent line to the curve at the point $P = (1, 3)$.

Solution:

Slope is $m = \frac{dy}{dx}$ evaluated at $P = (1, 3)$. Thus

$$m = -\frac{35(3)1^6}{3(3)^2 + 5(1^7)} = -\frac{105}{32}$$

The "Point-Slope Formula" for a line going through (x_o, y_o) with slope m is $y = m(x - x_o) + y_o$ so that the equation of the tangent to the curve at P is given by

$$y = -\tfrac{105}{32}(x - 1) + 3 = -\tfrac{105}{32}x + \tfrac{201}{32}$$

Additional Workspace: