Quiz 3 Solutions:

(1) $f(x) = \sin\left(\frac{\pi}{14}x\right) + \frac{42}{x} - 6$

* Is f cts?

Way 1:

$\left\{ \begin{array}{l} \sin\left(\frac{\pi}{14}x\right) \text{ cts everywhere} \\ \frac{42}{x} \text{ cts everywhere but } 0 \end{array} \right\}$

So, f is cts on $[7,14]$ since 0 is not in this set

Way 2: Let a be an arbitrary element of $[7,14]$

$$\lim_{x\to a} f(x) = \lim_{x\to a} \sin\left(\frac{\pi}{14}x\right) + \lim_{x\to a} \frac{42}{x} - 6$$

$$= \sin\left(\frac{\pi}{14}a\right) + \frac{42}{a} - 6 \quad \left(\text{since } a \neq 0\right)$$

So, f is cts at $x=a$ for all a in $[7,14]$ which means f is cts on $[7,14]$

* $f(7) = 1 > 0 > f(14) = -3$

Hence, by IVT, there is c in $(7,14)$ satisfying $f(c)=0$

but then c is a root of f

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(2) a) A.V: $\frac{s(5)-s(3)}{5-3} = \frac{\sqrt{28}-\sqrt{18}}{2}$

b) A.V: $\frac{s(3+h)-s(3)}{h} = \frac{\sqrt{18+5h}-\sqrt{18}}{h}$

c) I.V: $\lim_{h\to 0} \frac{s(3+h)-s(3)}{h} = \lim_{h\to 0} \frac{\sqrt{18+5h}-\sqrt{18}}{h}$

$$= \lim_{h\to 0} \frac{(\cancel{18}+5h)-\cancel{18}}{h(\sqrt{18+5h}+\sqrt{18})} = \lim_{h\to 0} \frac{5h}{(\sqrt{18+5h}+\sqrt{18})h}$$

$$= \lim_{h\to 0} \frac{5}{\sqrt{18+5h}+\sqrt{18}} = \frac{5}{2\sqrt{18}}$$

d) $y - \sqrt{18} = \frac{5}{2\sqrt{18}}(x-3)$

Extra: $f(x) = x^5 - 4x^3 - 3x + 1$ is cts everywhere since it's a polynomial.

$f(2) = -5 < 0 < f(3) = 127$

So, by IVT, f has a root c in the interval $(2,3)$.

$\boxed{\text{namely } f(c)=0}$