MA16204finaltry Glover, Kurt Name: _____

You must show your work. Answers with no work and/or no explanation will receive no credit. The point value of each problem is written to the left of the problem number. **We will grade only 8 out of 10 problems. You must clearly mark out the problems you do not want us to grade.**

(20)1. Graph $z^2 - x^2 - y^2 = 1$.

(20)2. Describe the volume inside of both $x^2 + y^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in cylindrical coordinates.

(20)3. Describe the volume inside both $x^2 + y^2 = z^2$ and $x^2 + y^2 + z^2 = 1$ in spherical coordinates.

(50)4. Find $\mathbf{v}$, $\mathbf{a}$, a_T, a_N, $\mathbf{T}$, $\mathbf{N}$, $\mathbf{B}$, κ for $\mathbf{r}(t) = \cos t\ \mathbf{i} + \sin t\ \mathbf{j} + t\ \mathbf{k}$. Also graph $\mathbf{T}$, $\mathbf{N}$, $\mathbf{B}$ for $t = \pi/2$ and graph the circle of curvature at this time.

(20)5. Find the equation of the plane through (1,2,3), (2,3,4) and (3,4,6) and graph it.

(20)6. Find the line of intersection of the planes $x + y + z - 1 = 0$ and $x - y = 0$ and graph it.

(20)7. Find the equation of the line tangent to the curve $\mathbf{r}(t) = \cos t\ \mathbf{i} + \sin t\ \mathbf{j} + t\ \mathbf{k}$ at $(0, 1, \pi/2)$.

2) $x^2+y^2=1$

x,y,z $\quad x^2+y^2+z^2=4$

↓ cyl.

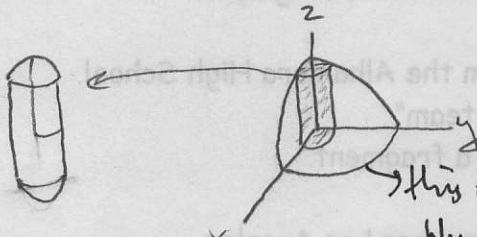
(r,θ,z) $\quad r^2=1 \to r^2\le 1$

$r^2+z^2=4 \to r^2+z^2\le 4$

$-1\le r\le 1$	$-2\le r\le 2$
$0\le\theta\le\pi$	$0\le\theta\le 2\pi$
$-\infty\le z\le\infty$	$-\sqrt{4-r^2}\le z\le\sqrt{4-r^2}$

$-1\le z\le 1$

$-\sqrt{1-z^2}\le r\le\sqrt{1-z^2}$



this is only the 1st quadrant ($x\ge 0, y\ge 0, z\ge 0$)

Intersection

$-1\le r\le 1$

$0\le\theta\le 2\pi$

$-\sqrt{4-r^2}\le z\le\sqrt{4-r^2}$

3) $x^2+y^2(+z^2)=z^2(+z^2)$

$x^2+y^2+z^2=1$

ρ^2

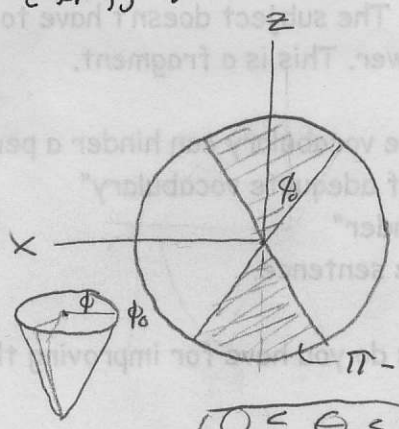
$\rho^2\le 1$ $\quad z=\rho\sin\phi$

$\rho^2-2\rho^2\sin^2\phi\ge 0$

$\rho^2(1-2\sin^2\phi)\ge 0$

$1-2\sin^2\phi\ge 0$

$0\le\sin\phi\le\frac{1}{2}$ ← $\sin^2\phi\le\frac{1}{2}$



because of symmetry we can just give a 2-dimensional picture. ~~Everything about y axis can be found by symmetry~~

$0\le\theta\le 2\pi$

$0\le\phi\le\phi_0$

$-1\le\rho\le 1$

$0\le\sin\phi\le\frac{1}{2}$

$0\le\phi\le\frac{\pi}{4}$ or $\frac{3\pi}{4}\le\phi\le\pi$

Hence we have

$(0\le\theta\le 2\pi)$

$(0\le\phi\le\pi/4$ or $\frac{3\pi}{4}\le\phi\le\pi)$

$(-1\le\rho\le 1)$