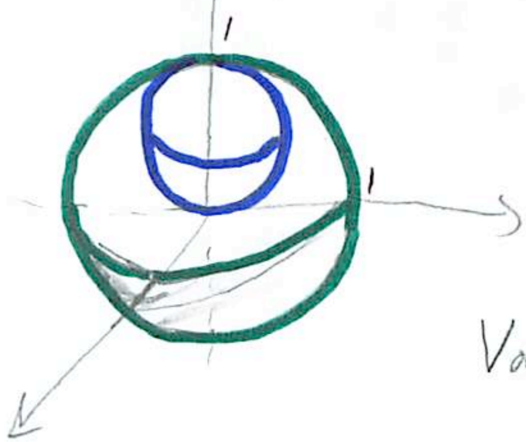
$\boxed{x^2+y^2+z^2=1}$ outside $\rho=1$

$\boxed{x^2+y^2+z^2=z}$ → in

inside $\rho^2=\rho\cos\phi \rightarrow \boxed{\rho=\cos\phi}$



We need to look at the volume in two parts. Lower semisphere and upper semisphere.

Volume of lower semisphere is

$$\frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{1}{2}\frac{4}{3}\pi(1)^3 = \boxed{\frac{2\pi}{3}}$$

(Volume of upper semisphere) − (blue sphere) $= \left(\frac{2\pi}{3}\right) - \left(\frac{4}{3}\pi\left(\frac{1}{2}\right)^3\right)$

$$= \frac{2\pi}{3} - \frac{\pi}{6} = \frac{3\pi}{6} = \boxed{\frac{\pi}{2}}$$

same

we can also find it by $\displaystyle\int_0^{2\pi}\int_0^{\pi/2}\int_{\cos\phi}^{1} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

$$= 2\pi \int_0^{\pi/2} \sin(\phi)\left(\frac{\rho^3}{3}\Big|_{\cos\phi}^{1}\right) d\phi = 2\pi \int_0^{\pi/2} \sin\phi\left(\frac{1}{3} - \frac{\cos^3(\phi)}{3}\right) d\phi$$

$u = \cos\phi$

$du = -\sin\phi \, d\phi$

$$= 2\pi \int_1^0 \left(\frac{1}{3} - \frac{u^3}{3}\right)\frac{du}{-1} = 2\pi\left(-\frac{1}{3}u + \frac{u^4}{12}\right)\Big|_1^0$$

$$= 2\pi\left(0 - \left(-\frac{1}{3} + \frac{1}{12}\right)\right) = 2\pi\left(\frac{3}{12}\right) = \boxed{\frac{\pi}{2}}$$ (THE SAME)

Volume $= \boxed{\frac{2\pi}{3} + \frac{\pi}{2}}$

Please, do read the second part carefully!!