MSLC – Math 151 AU 09
Exam 1 Review

Disclaimer: This review should NOT be used as your only guide for what to study.

PROBLEMS

1. What is the domain of tan(x)?

2. Use the laws of logarithms to rewrite the expression $\ln\left(x^5\sqrt{\frac{(y+7)^3}{z^9}}\right)$ in a form with no exponents.

3. Solve $e^{5x+1}=17$ for x

4. $f(x)=\dfrac{8x-5}{12x+1}$. Find $f^{-1}(x)$.

5. Find the domains of the following functions:

   a. $f(x)=\sqrt{\dfrac{x-1}{x+1}}$ b. $f(x)=\ln\left(x^2-4\right)$ c. $f(x)=\sqrt{1-\sqrt{9-x^2}}$

6. Given $f(x)=\sqrt{x}$ and $g(x)=x^2-5x+6$ find the functions:

   a) $h(x)=g\left[f(x)\right]$

   b) $k(x)=f\left[g(x)\right]$

   c) $l(x)=f\left[f(x)\right]$

7. Given $f(x)=\dfrac{6}{5-\sqrt{x-1}}$, find

   a. $f(12)$ b. domain of f c. all asymptotes of f d. $f^{-1}(x)$

8. Find the following limits:

a. $\lim\limits_{x\to 7^+}\dfrac{x^2-4x-21}{x^2-2x-35}$ b. $\lim\limits_{x\to 0^+}\dfrac{1}{1-e^{\sqrt{x}}}$ c. $\lim\limits_{x\to 6}\dfrac{|6-x|}{6-x}$

d. $\lim\limits_{x\to 0}\dfrac{\cos^2(x)-1}{\cos(x)-1}$ e. $\lim\limits_{x\to 1^-}e^{\frac{1}{x-1}}$ f. $\lim\limits_{t\to -2}\dfrac{7-t}{t^2-t-6}$

g. $\lim\limits_{x\to -\infty}\dfrac{2x^3-3x^2+4}{x^3+3x^2-1}$ h. $\lim\limits_{\theta\to\infty}\cos\theta$ i. $\lim\limits_{x\to -\infty}\dfrac{x^3}{\sqrt{4x^6-7}}$

j. $\lim\limits_{x\to 0}x^2\sin\left(\frac{1}{x}\right)$ k. $\lim\limits_{x\to 3}\dfrac{2x-\sqrt{3x^2+2x+3}}{x-3}$ l. $\lim\limits_{x\to\infty}2x-\sqrt{4x^2-x}$

9. Let $f(x)=\dfrac{x^3-2x^2-5x+6}{x^2+x-2}$. Find all discontinuities. Find the limits at the places where $f(x)$ is discontinuous.

10. Let $f(x)=\begin{cases} \dfrac{12}{x-3} & x<-1 \\ x+2 & -1\le x\le 3 \\ a-x & 3<x \end{cases}$ Find the following:

a. $f(-1)$ b. $f(3)$ c. $\lim\limits_{x\to 1} f(x)$ d. $\lim\limits_{x\to -1} f(x)$

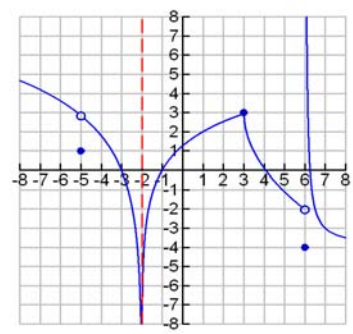
e. What value must a have in order for $f(x)$ to be continuous at $x=3$?

11. Let $f(x)=\dfrac{1}{x+1}$. Find $\lim\limits_{x\to 2}\dfrac{f(x)-f(2)}{x-2}$.

12. Let $f(x)=x^2-3x+2$. Using $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ find $f'(1)$. Give the equation of the tangent line at $x=1$.

13. Let $f(x)=\sqrt{x-4}$. Using $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ find $f'(6)$.

$f(x)$



14. Let $f(x)$ be the graph to the right. Find:

a) $\lim\limits_{x\to\infty} f(x)$ b) $\lim\limits_{x\to-\infty} f(x)$ c) $\lim\limits_{x\to -5} f(x)$

d) $\lim\limits_{x\to -2^-} f(x)$ e) $\lim\limits_{x\to -2^+} f(x)$ f) $\lim\limits_{x\to 3} f(x)$

g) $\lim\limits_{x\to 6^-} f(x)$ h) $\lim\limits_{x\to 6^+} f(x)$ i) $f(-5)$

j) $f(-2)$ k) $f(3)$ l) $f(6)$

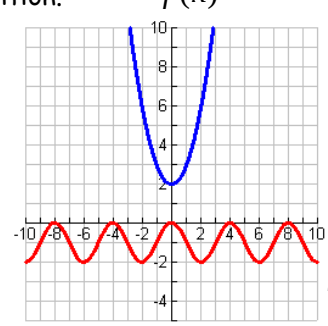
15. Sketch a graph with the following properties:

$\lim\limits_{x\to\infty} f(x)=4,\quad \lim\limits_{x\to-\infty} f(x)=+\infty,\quad \lim\limits_{x\to 4^-} f(x)=\infty$

$\lim\limits_{x\to 4^+} f(x)=-\infty,\quad \lim\limits_{x\to -3^-} f(x)=5,\quad \lim\limits_{x\to -3^+} f(x)=-2$

16. Use the Intermediate Value Theorem to prove that there exists a positive number c such that $c^2=2$. (Hint: Let $f(x)=x^2$. Choose your endpoints to be $x=1$ and $x=2$.) Clearly state each condition of the Intermediate Value Theorem and why this set-up satisfies the condition.

$f(x)$



$g(x)$

17. Let $f(x)$ and $g(x)$ be the functions given in the graph to the right. Find the following values:

a) $g(f(2))=$ b) $f(g(2))=$ c) $g(g(6))=$

18. Sketch the graph of $f(x)=\sqrt{x}$. Now sketch $f(x+2)$, $f(2x)$, and $-4f(x)$.

19. Use the derivative rules to find the derivatives of the following functions:

(a) $f(x)=x^{24}+7x^3$ (b) $g(x)=56e^x-8x^4$ (c) $h(x)=\dfrac{5}{x^3}+\sqrt[4]{x}$

Quick Summary of the Material

We strongly suggest you make your own study guide using your book and class notes. Here are a few things you may want to include:

1. A review of all the basic functions and their properties found in section 1.2 of your book.

2. The properties of exponential and logarithmic functions.

3. Odd/Even Functions:
   - An odd function is one that is symmetric about the origin. ex. $y = x$, $y = x^3$, $y = \sin x$
   - An even function is one that is symmetric about the y-axis. ex. $y = c$, $y = x^2$, $y = \cos x$

4. Domain & Range: Domain is the "input" into a function. Range is the "output" of a function.

5. Limits:
   - Intuitive definition – the limit L is the number the function approaches as x approaches the value a.
   - Limit Laws – Look in Section 2.3 for all of the limit laws.

6. The three essential parts of Continuity:
   A function $f(x)$ is continuous at $x = a$ (a need not be in the domain of $f(x)$) if
   1. $f(a)$ exists as a finite value
   2. $\lim_{x \to a} f(x)$ exists (which means $\begin{cases} \lim_{x \to a^-} f(x) \to L \\ \lim_{x \to a^+} f(x) \to M \end{cases}$ are both finite values and $L = M$)
   3. $f(a) = \lim_{x \to a} f(x)$

   If ALL three conditions hold, then $f(x)$ is continuous at $x = a$.

7. Average Rate of Change/Instantaneous Rate of Change:
   - The "average rate of change" is the rate of change of a function (or slope of the secant line) between two points $x = a$ and $x = b$ on a continuous function. This is calculated by $\frac{f(b) - f(a)}{b - a}$.
   - The instantaneous rate of change is the rate of change (or slope of the tangent line) that occurs at a single point, say $x = a$, of a function. It is calculated by $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ provided this limit exists. This is also called the derivative of the function at $x = a$, and we denote it $f'(a)$.

8. Intermediate Value Theorem:
   If $f(x)$ is continuous on the closed interval between the two points *a* and *b* on the *x*-axis, if *N* is a number that is between $f(a)$ and $f(b)$ on the *y*-axis, and if $f(a) \neq f(b)$, then there is number *c* between *a* and *b* on the *x*- axis such that $f(c) = N$.

9. This chart of special trig values:

Θ	0 (0)	π/6 (30)	π/4 (45)	π/3 (60)	π/2 (90)
Sin Θ	0	1/2	√2/2	√3/2	1
Cos Θ	1	√3/2	√2/2	1/2	0
Tan Θ	0	√3/3	1	√3	UND

10. Differentiation rules in section 3.1.

11. Anything the book has in a red box.