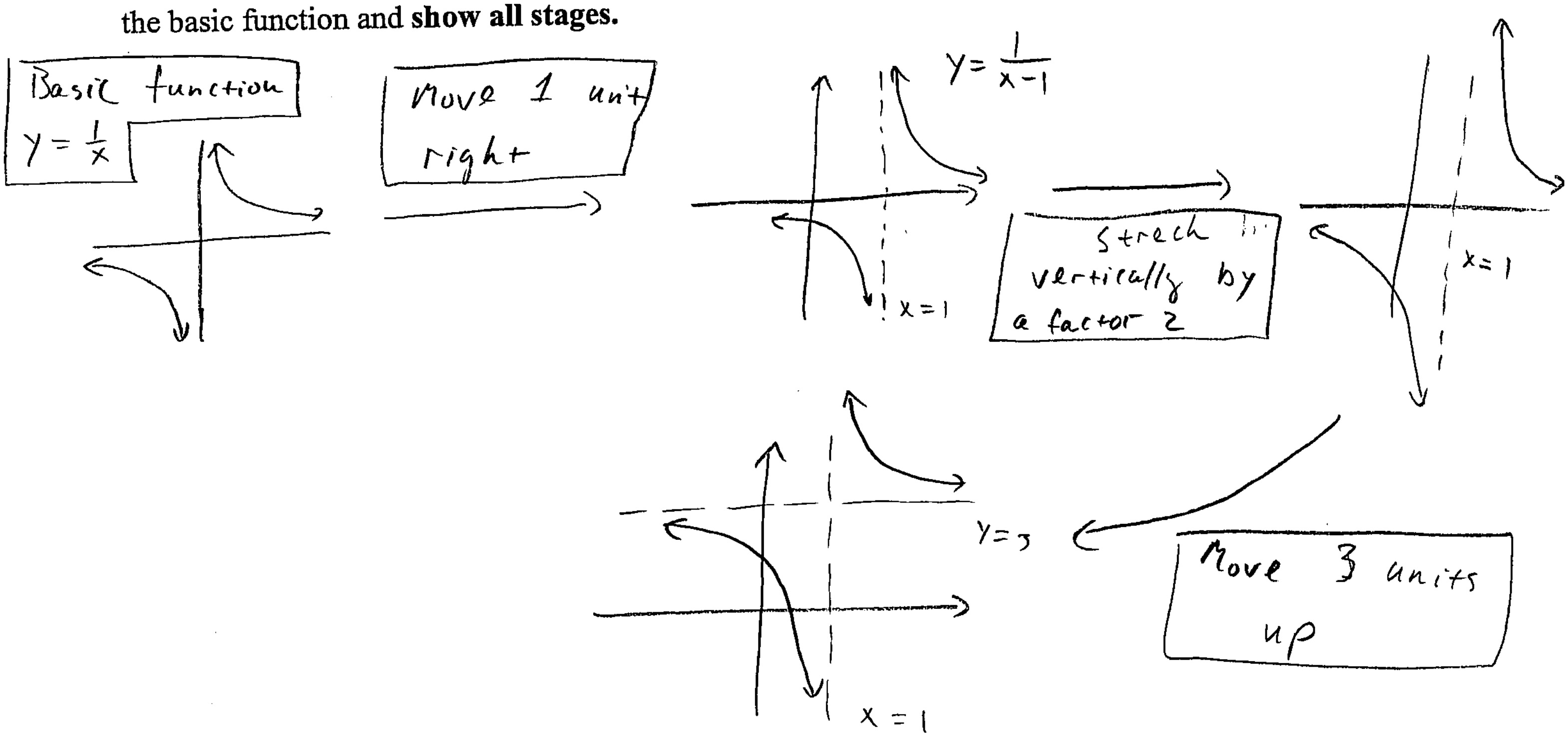
**Math 150** Sample Solution
Midterm 1
W 2008 1/31/08

Form B

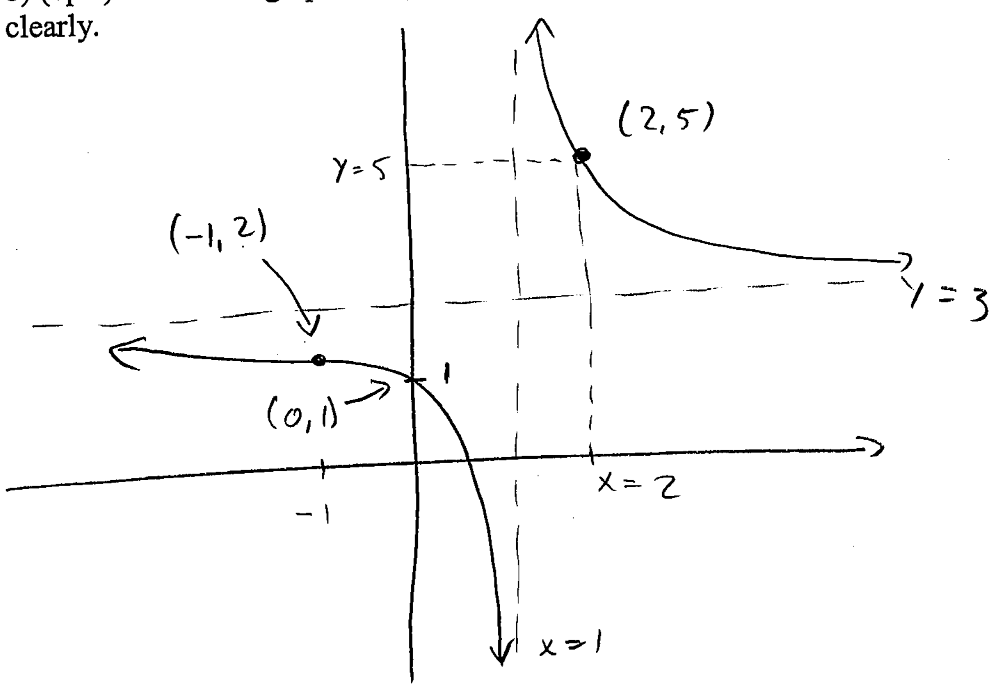
Name:____________________
Signature:____________________
Lecturer:____________________
Recitation Instructor:____________________
Recitation Time:____________________

Justify your answers by showing all of your work for each problem. If you use a graphing calculator, be sure to include detailed explanations of what you did. Partial credit will be awarded based on the work that is shown. Correct answers with no supporting work will **NOT** receive full credit. Please be neat and present your work in an organized manner. **Circle your answers.**

1. a) (10pts) Graph the function $f(x) = 3 + \frac{2}{x-1}$ using transformations. Start with the graph of the basic function and **show all stages.**




b) (6pts) Sketch the graph of f (x), plot at least three different points, and indicate their coordinates clearly.



2. Let $g$ be the function define by $g(x) = \dfrac{x^2 - 2x + 1}{2x^2 - 8x} = \dfrac{(x-1)\cdot(x-1)}{2x\cdot(x-4)}$

a) (6pts) Find x- and y-intercepts and vertical asymptotes.

+2 X-int. ⇒ $\boxed{x=1}$ i.e. $\boxed{(1,0)}$

+2 Y-int. ⇒ undef. !

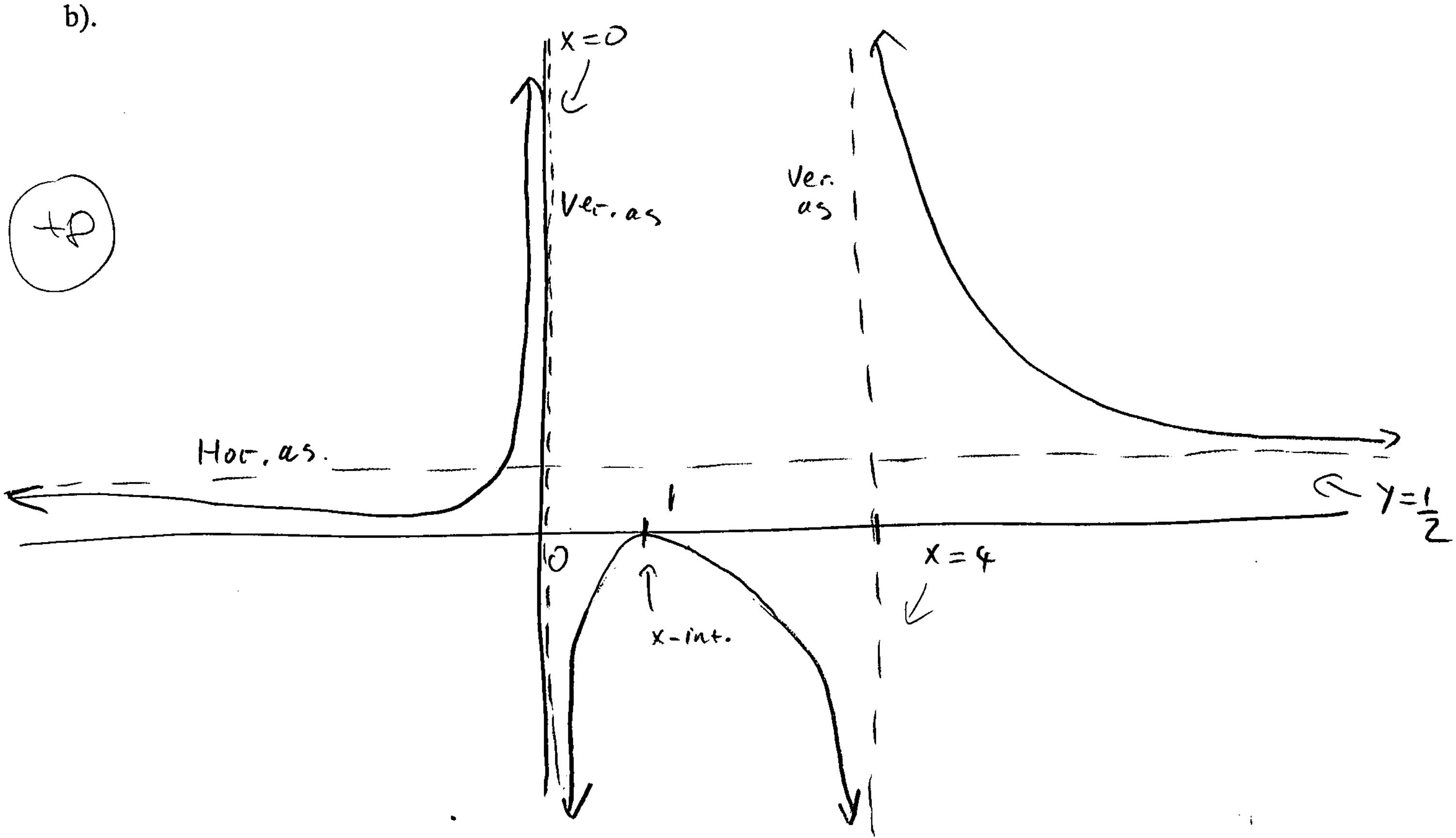
+2 Vert. asympt. ⇒ $\boxed{x=0}$ & $\boxed{x=4}$

b) (4pts) Find the horizontal or slant asymptotes.

+4 Hor. as. $\boxed{y = \frac{1}{2}}$

c) (8pts) Sketch the graph of $g$, making sure to include all the information found in parts a) and b).

+8



3. Let $P(x) = x^4 - x^2 + 2x + 2$

a) (10pts) Find all the zeros of P, real and complex.

By calc.: $\underbrace{P(-1) = 0 \quad - \text{mult. } 2}$

$\underbrace{\text{factors: } (x+1)\cdot(x+1)}$

$P(x) = (x+1)\cdot(x+1)\cdot(x^2 - 2x + 2)$

$$
\begin{array}{r}
x^2 - 2x + 2 \\
x^2+2x+1 \,\overline{)\, x^4 \qquad -x^2 + 2x + 2} \\
\ominus\; x^4 + 2x^3 + x^2 \qquad\qquad \\
\hline
-2x^3 - 2x^2 + 2x + 2 \\
\ominus\; -2x^3 - 4x^2 - 2x \qquad \\
\hline
2x^2 + 4x + 2
\end{array}
$$

$x + 1 = 0$ & $\underbrace{x^2 - 2x + 2 = 0}$

$\boxed{x = -1}$

via quad. formula

$\boxed{x = 1 \pm i}$

zeros of P : $\boxed{-1, -1, 1+i, 1-i}$

b) (6pts) Express P as a product of linear and irreducible quadratic factors with **real** coefficients.

$$\boxed{P(x) = (x+1)\cdot(x+1)\cdot(x^2 - 2x + 2)}$$

4. (16pts) Solve the inequality $\dfrac{2x+2}{2x-2} \geq \dfrac{2x+5}{x+1}$ algebraically.

$$\frac{2x+2}{2x-2} - \frac{2x+5}{x+1} \geq 0$$

$$\frac{x+1}{x-1} - \frac{2x+5}{x+1} \geq 0$$

$$\frac{(x+1)^2 - (2x+5)\cdot(x-1)}{(x-1)\cdot(x+1)} \geq 0$$

$$\frac{x^2+2x+1-2x^2-5x+2x+5}{(x-1)(x+1)} \geq 0$$

$$\frac{-x^2-x+6}{(x-1)(x+1)} \geq 0$$

$$\frac{-(x^2+x-6)}{(x-1)(x+1)} \geq 0$$

$$\frac{-(x+3)\cdot(x-2)}{(x-1)(x+1)} \geq 0$$

$$\boxed{\begin{array}{l} \text{Num.: } x=-3,\ x=2 \\ \text{den: } x=1,\ x=-1 \end{array}}$$

Set $R(x) = \dfrac{-(x+3)(x-2)}{(x-1)(x+1)}$

$(-\infty,-3)$   $-3$   $(-3,-1)$   $-1$   $(-1,1)$   $1$   $(1,2)$   $2$   $(2,+\infty)$   $\mathbb{R}$

| | $(-\infty,-3)$ | $(-3,-1)$ | $(-1,1)$ | $(1,2)$ | $(2,+\infty)$ |
|---|---|---|---|---|---|
| Test value | $-10$ | $-2$ | $0$ | $1.5$ | $+10$ |
| Sign of $R(x)$ | $R(-10) < 0$ | $R(-2) > 0$ | $R(0) < 0$ | $R(1.5) > 0$ | $R(10) < 0$ |
| where $R(x) \geq 0$ | ✗ | $[-3,-1)$ | ✗ | $(1,2]$ | ✗ |

$$\boxed{[-3,-1) \cup (1,2]}$$

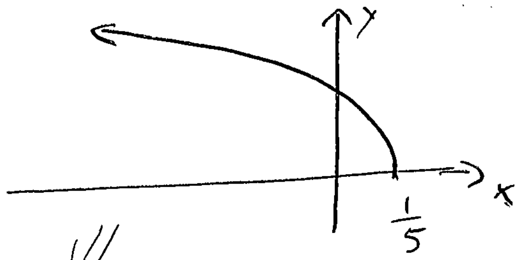
5. Consider the function $f(x)=\sqrt{1-5x}$

a) (9pts) Find the domain and range of *f* and explain why *f* is one-to-one.

$1-5x \geq 0$

$1 \geq 5x$

$\frac{1}{5} \geq x$




by Hor. line test

$\Downarrow$

$\boxed{f \text{ is one-to-one}}$ (+3)

$\Downarrow$

Domain $= (-\infty, \frac{1}{5}]$ (+3)

Range $= [0, +\infty)$ (+3)

b) (9pts) Find the inverse function $f^{-1}$ and its domain and range.

$y=\sqrt{1-5x}$

$y^2 = 1-5x$

$5x = 1-y^2$

$x = \frac{1}{5} - \frac{y^2}{5}$ $\quad (x \leftrightarrow y)$

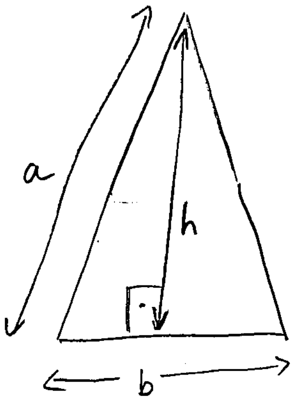
$y = \frac{1}{5} - \frac{x^2}{5}$

(+3) $f^{-1}(x) = \frac{1}{5} - \frac{x^2}{5}$

Domain: $[0, +\infty)$ (+3)

Range: $(-\infty, \frac{1}{5}]$ (+3)

6 (16pts) An isosceles triangle has a perimeter of 8cm. Find a functions that models its area A in terms of the length of its base b.




$$a^2 = h^2 + \left(\frac{b}{2}\right)^2$$

$$h^2 = a^2 - \frac{b^2}{4}$$

$$h = \sqrt{a^2 - \frac{b^2}{4}}$$

$$\text{Perimeter} = 8 = 2a + b$$

$$a = 4 - \frac{b}{2}$$

$$\text{Area} = \frac{b}{2} \times h$$

$$\text{Area} = \frac{b}{2} * \sqrt{a^2 - \frac{b^2}{4}}$$

$$\text{Area} = A = \frac{b}{2} \cdot \sqrt{\left(4 - \frac{b}{2}\right)^2 - \frac{b^2}{4}}$$

$$A = \frac{b}{2} \cdot \sqrt{16 - 4b + \frac{b^2}{4} - \frac{b^2}{4}}$$

$$\boxed{A = b \cdot \sqrt{4 - b}}$$