MIGHTY XLII

The 42^{nd} Midwest Graph Theory Conference

Ohio State University, Marion Campus

April 28-29, 2006

Invited Speaker	MR 200	11:00-12:00 am

On Degree Sequences Under Induced Subgraph Inclusion

Neil Robertson
Ohio State University

Let $D = (d_1, \ldots, d_n)$ and $D' = (d_1, \ldots, d_{n'})$ be the monotone degree sequences of simple finite graphs G, G', respectively. Define the partial order $D \leq D'$ when G, G' exist realizing D, D' and such that G is a vertex induced subgraph of G'. Around 1980 S. K. Rao conjectured that this is a well partial order on graphic degree sequences. This talk discusses known or conjectured properties, for fixed D, that may be forced on D' when $D \not\leq D'$. If true, these properties may be used to prove the Rao conjecture and may serve as rough models for the structure on G' when G is not an induced subgraph of G'.

Abstracts of Talks

Session III	MR 245	1:30-1:50 pm

Nowhere-zero 4-flow Petersen-minor free graphs

Taoye Zhang
West Virginia University

Tutte conjectured that every bridgeless graph without a Petersen minor admits a nowhere-zero 4-flow. Let $(P_{10})_{\bar{\mu}}$ be the graph obtained from the Petersen graph by contracting μ edges of a perfect matching. In this paper, we prove that every bridgeless $(P_{10})_{\bar{\mu}}$-minor free graph admits a nowhere-zero 4-flow for $\mu \geq 3$.

Session III	MR 245	1:55-2:15 pm

Every 3-connected claw-free Z_8 free graph is hamiltonian

Huiya Yan
West Virginia University

In this article, we first show that every 3-edge-connected graph with the circumference at most 8 is supereulerian, which is then applied to show that a 3-connected claw-free graph without Z_8

as an induced subgraph is hamiltonian, where Z_8 denotes the graph derived from identifying one endvertex of P_9 (a path with 9 vertices) with one vertex of a triangle. The above two results are both best possible in a sense that the number 8 can not be replaced by 9 and they also extend former results by Brousek, Ryjáček and Favaron in [Discrete Math. 196(1999)29-50] and by Łuczak and Pfender in [J. Graph Theory, 47(2004)111-121].

Session VI	MR 249	3:45-4:05 pm

Mod $(2p+1)$-orientations and $K_{1,2p+1}$-decompositions

Ju Zhou
West Virginia University

In this paper, we established an equivalence between the contractible graphs with respect to the mod $(2p+1)$-orientability and the graphs with $K_{1,2p+1}$-decompositions. Furthermore, it is applied to disprove a conjecture proposed by Barat and Thomassen.

Session I	MR 245	10:30- 10:50 am

Degree Sequences of F-Free Graphs

Oleg Pikhurko
Carnegie Mellon University

Let F be a fixed graph of chromatic number $r+1$. We prove that for all large n the degree sequence of any F-free graph of order n is, in a sense, close to being dominated by the degree sequence of some r-partite graph. As an application of our theorem, we present new results on the generalization of the Turán problem introduced by Caro and Yuster. This is a joint work with Anusch Taraz.

Session III	MR 245	2:20-2:40 pm

New sufficient condition for hamiltonian graphs

Zhao Kewen, Hong-Jian Lai, Yehong Shao*
Ohio University Southern

Let G be a graph and $\alpha(G)$ be the independence number of G. For a vertex $v \in V(G)$, $d(v)$ and $N(v)$ represent the degree of v and the neighborhood of v in G, respectively. In this paper, we prove that if G is a k-connected graph of order n, and if $\max\{d(v) : v \in S\} \geq n/2$ for every

independent set S of G with $|S| = k$ which has two distinct vertices $x, y \in S$ satisfying $1 \leq |N(x) \cap N(y)| \leq \alpha(G) - 1$, then G is Hamiltonian. This generalizes some former results by Dirac, Ore, Fan and Chen.

Session II	MR 249	10:30-10:50 am

Chord-Set Subgraphs

Terry McKee
Wright State University

A *chord-set subgraph* of a host graph H consists of all the chords of a cycle of H. After playing a bit with this very general notion, I will focus on the allegedly-natural special case when H is *chordal* (i.e., when the only induced cycles are triangles); but there must be more to the story than that ....

Session V	MR 245	3:45-4:05 pm

Spanning subsets of toroidal and Klein bottle embeddings

Chris Stephens and Xiaoya Zha*
Middle Tennessee State University

Let Φ be an embedding of graph G in a surface S. If there exists a subset K of S bounded by a subgraph of G which contains all the vertices of G, then K is called a *spanning* subset of Φ. Examples of spanning subsets include spanning discs, spanning annuli with some number of holes (called planarizing sets in some papers). A spanning subset may provide a simpler structure but still contain enough information to approach certain problems about graphs embedded on surfaces. In this talk, we discuss the existence of various spanning subsets in toroidal and Klein bottle embeddings. For example, we show that for any vertex in a 3-representative toroidal embeddings of a 3-connected graph there exists a spanning cylinder which contains the wheel neighborhood of that vertex.

Session III	MR 245	2:45-3:05 pm

Vertex identifying codes in graphs: definitions, theorems and open problems

Ryan Martin
Iowa State University

In 1998, Karpovsky, Chakrabarty and Levitin introduced a new graph invariant called the vertex identification code. If C is a subset of the vertices, then C is a **vertex-identifying code** if each set $N[v] \cap C$ is distinct and nonempty, where $N[v]$ denotes the closed neighborhood of vertex v. We will discuss a number of results on the size of the smallest code in a graph, particularly on the Erdős-Rényi random graph and we will present open problems.

Session I	MR 245	9:15-9:25 am

On 2-detour subgraphs of the hypercube

Jozsef Balogh
University of Illinois

A spanning subgraph H of a graph G is a 2-detour subgraph (or 2-additive spanner) of G if for each $x, y \in V(G)$, $d_H(x,y) \leq d_G(x,y) + 2$. Studying k-additive spanners was motivated by a number of problems in communication networks, broadcasting, routing, etc. We prove a conjecture of Erdős, Hamburger, Pippert, and Weakley by showing that for some positive constant c and every n, each 2-detour subgraph of the n-dimensional hypercube Q_n has at least $c \log n \cdot 2^n$ edges. This is joint work with A. Kostochka.

Session IV	MR 249	1:30-1:50 pm

Terrain Model acquisition by Robot Teams

Nachimuthu Manickam
DePauw University

The connectivity of the configuration space has been a valuable concept in the motion planning for single robots in both known and unknown terrains. We show here that n-connectivity plays a similar role for mobile robot teams in providing algorithms for terrain model acquisition in unknown terrains. A bound on the connectivity degree of the free space, reflected in that of a navigation course, provides us an estimate of the size of a robot team that is effective for the terrain. The robots are point-sized and equipped with visual sensors that acquire all visible parts of the terrain by scan operations executed from different locations. The performance is measured by the total number of scan operations performed by the robots. We employ the restricted visibility graph methods in a hierarchical setup. For terrains with convex obstacles and for teams of n (= 2, 3 or 4) robots, we prove that the total distance travelled is reduced by a factor of 1/n. For terrains with concave corners, the performance of the algorithm for the n (= 2, 3 or 4) robot team is expressed in terms of the sizes of n-connected components, and the sizes of the (n - 1)-or-less connected components.

Session V	MR 245	4:10-4:30 pm

Maximal Vertex-Connectivity of $\overrightarrow{A_{n,k}}$

Nart Shawash
Oakland University

Arrangement graph $A_{n,k}$ has a vertex set labeled by all the arrangements of k elements chosen from the ground set $\{1, 2, \ldots, n\}$. Two vertices are adjacent if their labels differ in exactly one of the k positions. $A_{n,k}$ contains both Star S_n and Alternating Group A_n graphs as special cases. $A_{n,n-1} \cong S_n$ which was proposed as an alternative to hypercube Q_n, while $A_{n,n-2} \cong A_n$. This talk presents modification to orientation of Arrangement graph previously given by Cheng and Lippman, and shows that a consequence of such an orientation is that unidirectional $A_{n,k}$ becomes maximally connected, that is $\overrightarrow{A_{n,k}}$ is $r-$connected, where $r = \min_{v \in V} \{\rho(v), \delta(v)\}$.

Session V	MR 245	5:00-5:20 pm

Counting Cycle Decompositions of 2-regular Digraphs

Bob Robinson
University of Georgia

A 2-regular digraph has indegree and outdegree 2 at every vertex. Loops and double arcs are allowed. It is elementary that every such digraph has a cycle decomposition, i.e., a partition of the arcs into directed cycles. Many 2-regular digraphs have more than one cycle decomposition. We show how to count the total number of cycle decompositions of all labeled 2-regular digraphs of given order n. The count can be also be obtained when the digraphs are restricted to having no loops or no double arcs.

Session V	MR 245	4:35-4:55 pm

Edge-Coloring of Multigraphs: Goldberg's Conjecture

Chris McClain
Ohio State University

In 1964, Vizing published his well known theorem stating that the chromatic index of a multigraph $\chi_E(G)$ is bounded above by $\Delta(G) + \mu(G)$, where $\Delta(G)$ is the maximum degree of a vertex in the graph and $\mu(G)$ is the maximum size of a parallel class of edges. In 1984 M.K. Goldberg republished in English some results he had published earlier in Russian, among them a refinement of Vizing's Theorem. He also conjectured that multigraphs whose chromatic index exceeds $\Delta + 1$ are *elementary* in some sense (which he defined precisely), and he proved the conjecture for a few small cases. In this talk, I will define Goldberg's notion of *elementary*, and I will explore a possible method of proof of his conjecture, motivated by Goldberg's proof of his partial result. This talk is based on work in progress.

Session IV MR 249 2:20-2:40 pm

Cost-Driven Transformation of Digraphs with Interdependent Nodes

Dave Anderson
Eymiha Corporation
Ann Darke
Bowling Green University

Computer applications are not what they used to be. With the advent of navigation-oriented technologies - such as the world wide web - applications are accessed as a network of linked pages that capture information as the user navigates between them and adds it to the state of the application, processing it upon demand. This network forms a graph that is amenable to analysis using Graph Theory.

When a correspondence is made between the navigation patterns of the application and in-order lists of visited nodes within the graph, it is possible to simplify the graph and transfer the simplifications back to the application domain, resulting in a "better" application. In our model these simplifications take the form of cost-driven transformations. There are many different kinds of transformations and many different costs that may be considered, the use and assignment of which depends on the application domain. We feel that when these transformations and costs mirror the costs of a real-world application's use and development, the simplifications transfered back to the application will be valid.

Session VI MR 249 4:35-4:55 pm

Bounds for the edge-bandwidth of the triangular grid

Reza Akhtar
Miami University

In 1995, Hochberg, McDiarmid, and Saks determined that the vertex-bandwidth of the triangular grid T_n was $n+1$; more recently, Balogh, Mubayi and Pluhár posed the question of determining the edge-bandwidth of T_n – that is, the vertex-bandwidth of $L(T_n)$.

I will discuss joint work with Tao Jiang, Zevi Miller, and Dan Pritikin in which we determine that the edge-bandwidth of T_n is bounded above by $3n-1$ and below by $3n-o(n)$.

Minimum p^{th} power domination in graphs

Matt Walsh
Indiana-Purdue University Fort Wayne

A fractional dominating function (FDF) on a graph G is an assignment of weights between 0 and 1 to its vertices so that the sum of the weights over each closed neighborhood is at least 1. A minimum pth power (fractional) dominating function on a graph is an FDF on the graph such that the sum of the p^{th} powers of the weights of all the vertices is a minimum; when $p = 1$, these are called minimum fractional dominating functions, and the minimum achieved is the fractional domination number $\gamma_f(G)$ of the graph. We investigate the corresponding p^{th} power parameters $\gamma_f^{(p)}(G)$ and the FDFs that achieve these values. This is joint work with Dean Hoffman, Peter Johnson and Robert Rubalcaba.

k-dependence and $\frac{1}{2}$-domination in kings graphs

Dan Pritikin
Miami University

Given an integer k, what is the maximum number of kings that can be placed on an n by n board (usually a toroidal board for us), no king adjacent to more than k kings? In graph theoretic terms this is the same as asking for the k-dependence number of the n by n kings graph. The cases $k = 4, 5$ are the most interesting ones. We allow the board to be n by n by ... by n with arbitrarily many dimensions. Linear programming and balanced ternary notation come into play. This is joint work with Eugen Ionascu.

Bounding Anti-Ramsey numbers via Turán numbers

Tao Jiang
Miami University

Let n be a positive integer and $\mathcal{H}$ a family of graphs. The *Turán number* $ex(n, \mathcal{H})$ is the maximum number of edges of an n-vertex graph that does not contain any member of $\mathcal{H}$ as a subgraph. The *Anti-Ramsey number* $AR(n, \mathcal{H})$ is the maximum number of colors in an edge-coloring of K_n that does not contain any member of $\mathcal{H}$ all of whose edges are colored differently. These two functions are closely related.

Given a graph H, let $\mathcal{H}^-$ denote the family of graphs obtainable from H by removing a single edge. Erdős, Simonovits, and Sós showed that always $1 + ex(n, \mathcal{H}^-) \leq AR(n, H) \leq ex(n, \mathcal{H}^-) + o(n^2)$, where the lower bound is achieved when $H = K_p$. Later results by various authors indicated that the lower bound is within $O(n)$ from the actual value of $AR(n, H)$ when H is a tree, a cyle, or the complete bipartite graph $K_{2,m}$.

Here, we sharpen the upper bound above for some special classes of graphs to show that the lower bound is tight or asymptotically tight for those classes of graphs. In the first part, we show that if H is a subdivided graph (a graph obtained from another graph by subdividing each edge once) then $AR(n, H) \leq ex(n, \mathcal{H}^-) + O(n)$. In the second part, we show that if H is a graph such that $\chi(H - e) \geq p + 1$ for each edge $e \in E(H)$ and there exist two edges e_1, e_2 of H for which $\chi(H - e_1 - e_2) = p$, then for sufficiently large n, $AR(n, H) = 1 + ex(n, \mathcal{H}^-) = t(n, p) + 1$ or $t(n, p) + p$, where $t(n, p)$ denotes the number of edges in the Turán graph $T_{n,p}$. This yields one of the very few exact results on Anti-Ramsey numbers. The proof uses the first stability theorem of Simonovits for the Turán problem.

The work in the second part of the talk is joint with Oleg Pikhurko.

Session VI MR 249 5:00-5:20 pm

The edge-bandwidth of Hamming graphs

Zevi Miller
Miami University

The edge-bandwidth $B'(G)$ of a graph G is the bandwidth of the line graph of G. The Hamming graph K_n^d is the d-fold Cartesian product of the complete graph K_n. We obtain lower and upper bounds on the edge-bandwidth of the Hamming graph. For fixed even n, our lower and upper bounds match asymptotically as functions of d, showing that

$$B'(K_n^d) \sim \sqrt{\frac{d}{2\pi}} \cdot n^d \cdot (n-1).$$

This extends a recent result of Balogh, Mubayi, and Pluhár that $B'(K_2^d) \sim \sqrt{\frac{d}{2\pi}} \cdot 2^d$.

If time allows, we will also discuss the edge-bandwidth of the mutli-dimensional grids P_n^d. For every fixed d, we determine $B'(P_n^d)$ asymptotically as a function of n, showing that

$$B'(P_n^d) \sim c_d \cdot n^{d-1},$$

where c_d is a constant depending solely on d. For instance, $c_2 = 2$ and $c_3 = \frac{9}{4}$, and so on. This extends a result of Balogh, Mubayi, and Pluhár that $B'(P_n^2) \sim 2n$.

The results presented in this talk are joint with Reza Akhtar and Tao Jiang.

Session IV MR 249 1:55-2:15 pm

A Graph Editor and Graph Algorithms Implementation System

Jay Bagga and Sunita Upadrasta
Ball State University

JEdit is a Java based system for drawing graphs and running graph algorithms. This system is basically designed to be a teaching and learning tool. The extensive graphical user interface includes various options for creating and manipulating graphs.

With JEdit we can run graph algorithms with animation, draw special graphs with just a click, and perform simple graph operations. Some algorithms can be run with an animation feature where the user can see intermediate steps as the algorithm executes.

The new version of JEdit includes extensive documentation of classes and an interface for easy addition and removal of algorithms. Several standard algorithms are also included.

Session IV	MR 249	2:45-3:05 pm

The Disk Covering Problem Revisited

Santosh Kumar
Department of Computer Science and Engineering
Ohio State University

It is well-known that placing disks (each of radius r_s) in the triangular lattice pattern is optimal for covering all the points on a plane (rigorously proved by R. Kershner in 1939). We consider the following variation on the problem: Suppose we additionally want the centers of the disks to form a connected network, where two disk centers are said to be directly connected if they are within an Euclidean distance of r_c from each other. If $r_c/r_s \geq \sqrt{3}$, then the triangular lattice pattern is still optimal for achieving coverage and connectivity both. What if $r_c/r_s < \sqrt{3}$? In this work, we find an optimal pattern that provides both coverage and connectivity for general values of r_c/r_s and prove its optimality. This problem has become important because of its applications to the deployment of wireless sensor networks.

This is a joint work with Xiaole Bai, Dong Xuan, Ziqiu Yun, and Ten H. Lai. This work has been accepted to appear in the proceedings of the *ACM MobiHoc 2006* conference under the title "Deploying Wireless Sensors to Achieve Both Coverage and Connectivity."

Session I	MR 245	10:05-10:25 am

Rao's Conjecture

Chris Altomare
Ohio State University

Rao's Conjecture states that degree sequences of graphs are well-quasi-ordered under a certain natural notion of containment involving the induced subgraph relation. In this talk, we'll discuss partial progress toward a solution, including the plan of attack developed by Micu, Song, and Robertson and a new proof of Rao's conjecture for degree sequences of bounded degree found by the speaker.

Session I	MR 245	9:40-10:00 am

Chromatic cohomology theory for graphs

Sergei Chmutov
The Ohio State University Mansfield

Recently Laure Heleme-Guizon and Yongwu Rong from George Washington University introduced a cohomology theory for graphs where the graded Euler characteristic is the chromatic polynomial of the graph. As a consequence this theory gives a new description of coefficients of the chromatic polynomial.

Session II	MR 249	10:05-10:25 am

Some new results on packing of sparse graphs

Gexin Yu
University of Illinois at Urbana-Champaign

Two n-vertex graphs G_1 and G_2 pack if there exist injective mappings of their vertex sets into $[n]$ such that the images of the edge sets do not intersect. Recently, packing of sparse graphs attracted considerable attention.

In this talk, we present some new results on packing graphs under given degree conditions.

The talk is based on joint work with H. Kaul and A. Kostochka.