Here are a few problems for the $SL(2,\mathbb{R})$-class.

Exercise (M): Let $\gamma_0(t) = a^t i$ for $t \in [0,1]$ and some fixed $a > 0$. Show that this path has length $L(\gamma_0) = |\ln a|$, and that there is no shorter piecewise smooth path connecting i and ai.

Exercise (E): Check that our map $SL(2,\mathbb{R}) \times \mathbb{H} \to \mathbb{H}$ is indeed an action.

Exercise (E): Verify that $\frac{\|v\|}{Im(z)} = \frac{\|\frac{1}{(cz+d)^2}v\|}{Im(g(z))}$, i.e. that the hyperbolic length of a tangent vector is unchanged under the action.

Exercise (M): Show that $d(x_0,x_1) = \inf_\gamma L(\gamma)$ is a metric and $SL(2,\mathbb{R})$ acts by isometries. Here the infimum is taken over all continuous piecewise smooth paths connecting x_0 and x_1.

Exercise (E): Verify all (remaining) claims of the $PSL(2,\mathbb{R})$-theorem.

Exercise (E): Use the Measure-transitivity lemma to prove that $f(x) = f(gx)$ for m-a.e. x (where the null set may depend on g) and all g implies for a measurable function f that $f(x) = const$ for m-a.e. x.

Exercise (E): Prove that $SL(2,\mathbb{R})$ is generated by the upper and lower unipotent matrices.

Exercise (M with hint, H without):a) Prove that $\Gamma = \{\gamma \in SL(2,\mathbb{Z}) : \gamma \equiv I \text{ modulo } 2\}$ is a lattice in $SL(2,\mathbb{R})$ and that $\Gamma/\{\pm I\}$ is the free group generated by $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$.
b) Prove that $PSL(2,\mathbb{Z})$ is the free product of the cyclic group of order 2 generated by $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ and another cyclic group of order 3.

Exercise (M): Let $\Gamma \subset G$ be a lattice. Then the induced measure on $X = \Gamma\backslash G$ is right G-invariant.

Exercise (E): Show that $G = \left\{ \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} \right\}$ cannot have a lattice.