

MAY 14, 2009

- (1, E) Show that the full group of automorphisms of a d -ary tree is uncountable.
- (2, E) Identify the groups $G^{(2)}/St_{G^{(2)}}(k)$ of automorphisms of binary trees of depth k for the values $k = 1, 2, 3$.
- (3, O) Does there exist a self-similar group with superpolynomial complexity of the word problem?
- (4, M) Define composition of automata, figure out what the inverse is for an invertible automaton A , and describe its Moore diagram.
- (5 E –; D) Describe some automata with 2 states over the alphabet with 2 letters and groups generated by them.
Classify all groups generated by such automata.
- (6, M) Describe a self-similar action of a free abelian group.
- (7, E-M) The first Grigorchuk group is defined as $\Gamma = \langle a, b, c, d \rangle$ with $a = (1, 1)\epsilon$, $b = (a, c)e$, $c = (a, d)e$, $d = (1, b)e$. Show that it is a quotient of the free product of a group of order 2 with the 4-element Klein group V .
- (8, D) Show directly that for each n there exists $\gamma \in \Gamma$ such that $\gamma^{2^n} \neq 1$. (Hint: consider the subgroup $K < \Gamma$, the normal closure of $(ab)^2$.)
- (9, O) Does there exist a group generated by a finite automaton with unsolvable conjugacy problem?