

Correction:
The Polar Dual of a Convex
Polyhedral Set in Hyperbolic Space

RUTH CHARNEY & MICHAEL DAVIS

The “polar dual” $P(X)$ of a convex polyhedral set X in hyperbolic space is defined twice in [1]. In fact, as B. Okun has pointed out, the two definitions don’t always coincide in the case where X has infinite volume.

The first definition is given in the first paragraph of the Introduction: $P(X)$ is the set of outward-pointing unit normal vectors to the supporting hyperplanes of X , that is,

$$P(X) = \{ u \in \mathbb{S}_1^n \mid \langle u, x \rangle \leq 0 \text{ for all } x \in X \text{ and } \langle u, x \rangle = 0 \text{ for some } x \in X \}.$$

(This definition is repeated on p. 493 at the beginning of Sec. 3.1.) This is the correct definition and is the one used in proving the main theorem in Sections 3 and 4. The second definition [1, p. 489] is given as: $P(X) = C_{>0}^* \cap \mathbb{S}_1^n$, where $C_{>0}^*$ is defined as the union of the spacelike faces of the dual cone C^* .

The problem is that a timelike face of C may fail to intersect \mathbb{H}^n (it may intersect the other sheet of the hyperboloid instead), in which case not every spacelike face of C^* is dual to a face of X . To make the two definitions agree, we must take $C_{>0}^*$ to consist only of those faces of C^* that are dual to faces of X . The remainder of Section 2 then holds, providing we replace the phrase “timelike face of C ” by “timelike face of C that intersects \mathbb{H}^n ”.

Reference

- [1] R. Charney and M. Davis, *The polar dual of a convex polyhedral set in hyperbolic space*, Michigan Math. J. 42 (1995), 479–510.